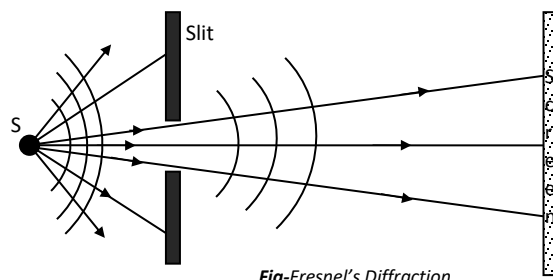


DIFFRACTION

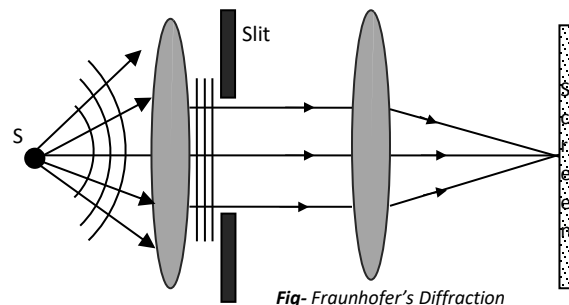
The bending(spreading) of light around the corners(edge) of the narrow opening or obstacles is called *diffraction*.

Fresnel's Diffraction:

In this type of diffraction, the source of light or screen or both are at finite distances from obstacle or aperture. No lenses are used to make the rays parallel or convergent. The incident wave front is not plane but either spherical or cylindrical



Fraunhofer's diffraction: In this type of diffraction, the source of light and the screen are effectively at infinite distances from the aperture or obstacles. Two convex lenses are used, one to before it falls on the narrow



aperture and other to focus the light after diffraction on the screen. This arrangement in fact removes the sources and the screen to infinity.

Fraunhofer's Diffraction at a Single Slit

Let Consider a narrow slit AB of width 'd' illuminated by a parallel monochromatic beam of light of wavelength λ . The light passing through the slit suffers

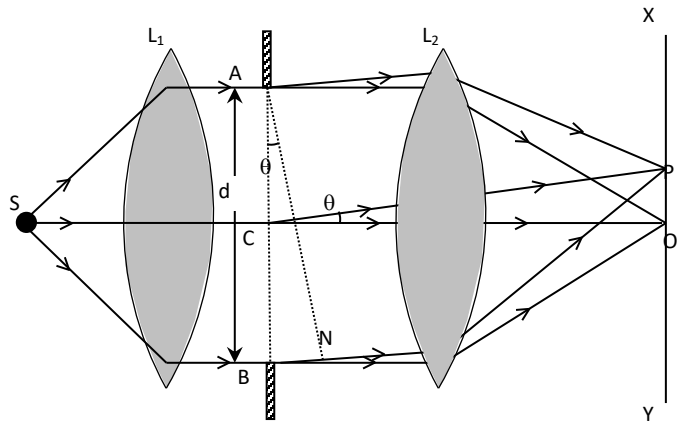


Fig Fraunhofer's Diffraction at a single slit

diffraction. If a lens L_2 is placed in the path of the diffracted beam, a real image of diffraction pattern is obtained on the screen XY placed perpendicular to the plane of paper in the focal plane of the lens. When a plane wavefront is incident on the slit AB, each point on the wave front becomes a source of secondary wavelets on the basis of Huygen's theory.

Central maximum

The secondary wavelets travelling in a direction parallel to CO comes to focus at O and a bright central image is formed. This is due to the fact that the secondary waves

from points equidistant from 'C' and situated in the upper and lower halves CA & CB of the wavefront will travel same distance in reaching at 'O' which results the path-difference to be zero. Thus, point 'O' is the position of maximum intensity which is known as the central maximum.

Now, suppose the secondary waves travelling in a direction making an angle θ with CO. These secondary wavelets are brought to focus by the lens L_2 at a point 'P' which will have a maximum or minimum intensity depending upon the path - difference between the secondary waves originating from the corresponding points of the wave front.

Position of secondary minima: Draw AN perpendicular to the direction of the diffracted rays from A.

$$\text{In } \triangle ABN, \quad \sin \theta = \frac{BN}{AB}$$

$$\therefore BN = AB \sin \theta = d \sin \theta$$

Where BN is the path - difference between the wavelets from the extremities of the slits

wavefront AB can be considered to be made up of two halves AC & BC. If the path - difference between secondary

wavelets from A & B is λ , then the path - difference between the secondary waves from A & C is $\frac{\lambda}{2}$. Similarly, the path - difference between B & C is $\frac{\lambda}{2}$. Hence, it gives rise to destructive interference at P which has minimum intensity.

For 1st minimum, $BN = d \sin \theta = \lambda$

If the path - difference between the extreme wavelets from A to B is 2λ ,

i.e. $BN = d \sin \theta = 2\lambda$

Then the intensity will again be minimum because the slit can now be supposed to be divided into four equal parts and the rays from corresponding points separated by a distance $\frac{d}{4}$ in the two halves of each half of the slit will

have a path - difference of $\frac{\lambda}{2}$, thus .

Hence, For 2nd minimum, $BN = d \sin \theta = 2 \lambda$

In general, for the various secondary minima, the path - difference BN between the extreme wavelets from A & B

should be an integral multiple of λ . Thus, for secondary minima, we have

$$d \sin \theta_n = n\lambda$$

Where, $n = 1, 2, 3, \dots$ and the angle θ_n gives the direction of the n^{th} minimum.

Position of secondary maxima

In addition to the central maximum at 'O' there are secondary maxima which lie in between the secondary minima on either side of the central maximum. These are situated in a direction on which the path - difference BN is an odd multiple of $\frac{\lambda}{2}$. Thus, for secondary maxima

$$(2n + 1) \frac{\lambda}{2} = d \sin \theta_n$$

Where, $n = 1, 2, 3, \dots$ and thus, we find that the diffraction pattern due to a single slit consists of a central maximum at 'O' followed by secondary minima and maxima on both sides.

The intensity distribution of the diffraction pattern due to a single slit as a function of ' θ ' is shown in Fig-

The intensity of secondary maxima goes on decreasing rapidly as seen from the curve.

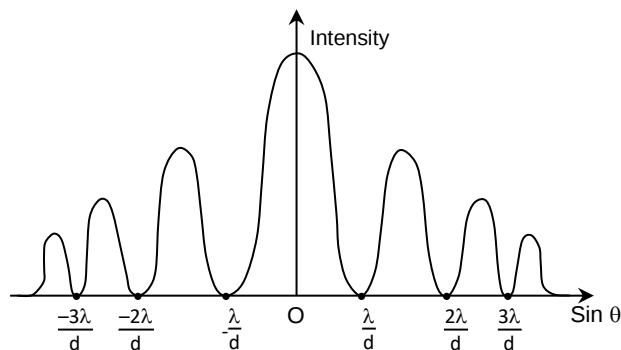


Fig- Intensity distribution in Fraunhofer's diffraction

Width of Central maximum:

Width of the central maximum is defined as the distance between the first minima on either side of the central maximum 'O'. Referring from Fig-the first secondary minimum occurs at point P_1 & P_2 . So.

$$d \sin \theta = n \lambda$$

For 1st minimum, $n = 1$, and $\sin \theta = \frac{\lambda}{d} \Rightarrow \theta = \frac{\lambda}{d}$ [Since $\sin \theta \approx \theta$, for small θ]

The angle subtended by the whole central maximum is 2θ . Thus, we get

$\therefore$ Angular width of central maximum $(2\theta) = \frac{2\lambda}{d}$ (i)

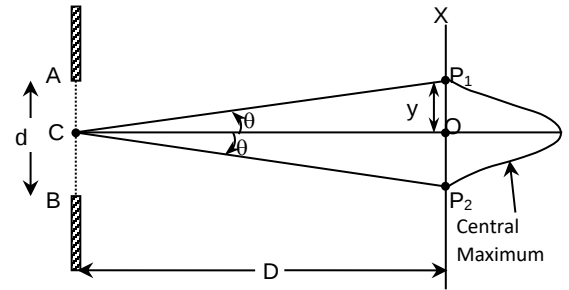


Fig- Width of central maximum

This gives the angular width of the central maximum.

If y is the distance from point 'O' to the first minimum on either side of central maximum, then width of central maximum is $2y$.

Let 'D' be the distance between the slit AB and the screen XY.

Now, in $\triangle COP_1$, $\tan \theta = \frac{y}{D}$

Since θ is small angle, $\tan \theta \approx \theta$, $\theta = \frac{y}{D}$

or, $2\theta = \frac{2y}{D}$ (ii)

From Eq. (i) & (ii), $\frac{2y}{D} = \frac{2\lambda}{d} \Rightarrow \therefore 2y = \frac{2\lambda D}{d}$

Hence, width of central maximum is,

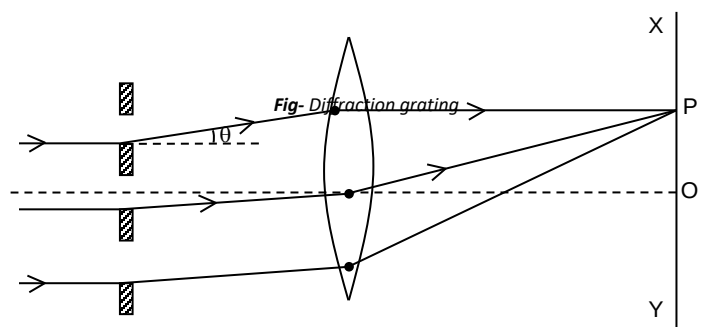
$\therefore$ Linear width of central maximum $(\beta) = 2y = \frac{2\lambda D}{d}$

Diffraction Grating

An arrangement which consists of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called diffraction

grating. It is constructed by ruling equidistant parallel lines on a transparent material, such as glass plate by means of a fine diamond point. The ruled lines act like opaque and thus do not allow the incident light to pass through them. The space in between the lines are called transparencies.

Let 'a' and 'b' be the width of transparencies and opacities



respectively. The value $(a + b)$ is known as grating element. If N is the number of lines per inch of the grating, then grating element is given by

$$a + b = \frac{1}{N} \text{ inch} = \frac{2.54}{N} \text{ cm}$$

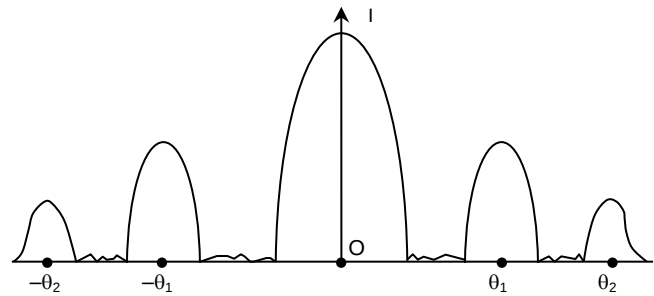


Fig-Intensity distribution in diffraction grating

Theory: Consider a Parallel beam of monochromatic light (i.e. Plane wave) of wavelength ' λ ' is incident normally on the grating surface. Let $(a + b)$ be the grating space or grating element. The

secondary maxima with multiple slits occurs into the same direction as for the two - slit pattern in young's experiment. Let θ_1 is angle of diffraction for first secondary maximum. We have,

$$d \sin \theta_1 = n\lambda$$

$(a + b) \sin \theta_1 = \lambda$, Where $n = 1$ & $d = a + b$.

Similarly, for 2nd order maximum ($n = 2$), we get $(a + b) \sin \theta_2 = n\lambda$

In general, for n^{th} order maximum,

$$(a + b) \sin \theta_n = n\lambda, \text{ Where } n = 1, 2, 3, \dots$$

When $n = 0$, we get the central maximum at 'O'. Intensity variation of diffraction patterns on the screen against ' θ ' is shown in Fig. The central principle maximum is the most intense & gradually decreases on its either sides

Resolving power

The resolving power of an optical instrument is defined as its ability to resolve or separate the images of two nearby point objects so that they can be distinctly seen.

Resolving power of a microscope: The resolving power of a microscope is defined as reciprocal of the smallest distance between two point objects at which they can be just resolved when seen through the microscope.

The smallest distance between two point object at which they can be just resolved by the microscope, or the limit of resolution, is given by;

$$d = \frac{\lambda}{2\mu \sin\theta}$$

Where, λ = the wavelength of light used,

θ = half the angle of cone of light from each point object or the angle subtended by each point object

or the angle subtended by each point object on the radius of the objective

μ = the refractive index of the medium between the point object and the objective of the microscope.

$$\therefore \text{Resolving power of a microscope} = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

Resolving power of a telescope: The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images can be just resolved by it. The smallest linear angular separation between two distant objects whose images can be just resolved by the telescope, or the limit of resolution, is given by

$$d\theta = \frac{1.22\lambda}{D}$$

Where, λ = the wavelength of light,

D = the diameter of the telescope objective, and

$d\theta$ = the angle subtended by the two distant objects at the objective.

Numericals

1. How wide is the central diffraction peak on a screen 3.5 m behind a 0.010 mm slit illuminated by 500 nm. light? Ans: 0.35 m
2. A Parallel beam of sodium light of wavelength $5.893 \times 10^{-7} \text{m}$ is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is 28° . Find the number of ruling lines per mm on the grating.
Ans: 406 lines/mm
3. A diffraction grating has 400 lines per mm and is illuminated normally by a monochromatic light of wavelength $6000 \text{ \AA}$. Calculate the grating spacing, the angle at which first order maximum is seen and the maximum number of diffraction maxima obtained.
Ans: $0.25 \times 10^{-5} \text{m}$, 13.90 , $n = 4$
4. A parallel beam of sodium light of wavelength 589.3 nm is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is $27^\circ 42'$. What will be the number of lines per mm on the grating?
Ans: 406 lines/mm
5. A parallel beam of sodium light of wavelength 589.3 nm is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is $27^\circ 42'$. What will be the number of lines per mm on the grating?

Ans: 406 lines/mm

6. A parallel beam of sodium light is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is $27^\circ 42'$. Assuming that the wavelength of light is 589.3nm. Find the number of lines per mm on the grating.

Ans: 406lines/mm

7. A plane transmission grating having 500 lines per mm is illuminated normally by light source of 600 nm wavelength. How many diffraction maxima will be observed on a screen behind the grating.

ANS 3

Short questions

1. Diffraction grating is better than a two-slit set up for measuring the wave length of a monochromatic light. Explain.
2. Radio waves diffract around building but light waves do not. Why?
3. The diffraction of sound waves is more evident in daily experience than that of light waves, why?
4. Describe what happens to the single slit diffraction pattern when the width of the slit is less than the wave length of the wave.
5. What are the characteristic elements associated with a diffraction grating? How is plane transmission grating constructed?
6. What is the fundamental physical difference between interference and diffraction? Explain with figures.
7. Why can we readily observe diffraction effects for sound waves but not light?
8. Light waves undergo diffraction around an edge. Can sound wave diffract around an edge? Explain.

