Mechanical Wave

The waves which can be produced or propagated only in the material medium are called mechanical waves.

Speed of Sound Wave in Any Medium

As sound wave needs a material medium for its propagation and medium must possesses two physical properties: elasticity and inertia, the wave velocity obviously depends on these properties. Thus the velocity of sound wave in any medium depends on the elasticity (E) of the medium and its density (ρ). Let v is the velocity of the wave, then we have

$$v \propto E^x \rho^y$$

or, $v = K E^x \rho^y$ ....(i)

Where K is a proportionality constant, x and y are indices to be determined.

Now, the dimensional formula of v = $[LT^{-1}]$

The dimensional formula of E = $[ML^{-1} T^{-2}]$

The dimensional formula of ρ = $[ML^{-3}]$

From dimensional analysis, from Eq.(2.1), we get

$$[LT^{-1}] = [ML^{-1} T^{-2}]^x [ML^{-3}]^y$$

or, $$[LT^{-1}] = [M^{x+y} L^{-x-3y} T^{-2x}]$$

Eqating the indices of [M], [L] and [T] repectively on both sides ,we get

$$x + y = 0 \quad , \quad -x - 3y = 1 \quad \text{and} \quad -2x = -1$$

on solving these equations , we get

$$x = \frac{1}{2} \quad \text{and} \quad y = \frac{-1}{2}$$

Substituting these values of x and y in Eq.(i), we get

$$v = K E^{\frac{1}{2}} \rho^{\frac{-1}{2}}$$

or, $v = K\sqrt{\frac{E}{\rho}}$

From mathematical analysis, we get K = 1 and the velocity of sound becomes , $v = \sqrt{\frac{E}{\rho}}$

Hence , speed of sound in any medium can be generalized as,

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\text{Elastic factor}}{\text{Inertial factor}}}$$

Speed of Sound in solid

Since solid has different shape and size, it possesses different modulus of elasticity and the relations for velocity of sound for the solid of different shapes are different. Also nature of the vibration of the particles of the medium affects the speed of wave propagation.

(a) Speed of longitudinal wave in solid

In solid rod , the Young's modulus of elasticity is taken into account. Hence speed of sound in solid rod is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

In solid of other shape : For the bulk solid(rather than thin rod), the bulk modulus (K) and the modulus of rigidity (η) play role for propagation of sound wave and hence speed of longitudinal wave is given by,

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

(b) Speed of transverse wave in solid

The mechanical waves travelling in solids as transverse waves are called ***shear-waves*** and their speed is given as,

$$v = \sqrt{\frac{\eta}{\rho}}$$,where η = modulus of rigidity

Also for the transverse mechanical waves propagating along a string, their speed is given by the relation,

$$v = \sqrt{\frac{T}{\mu}}$$

Speed of Sound in Liquid

In case of liquids, the modulus of rigidity is negligible and the bulk modulus (K) becomes significant. Hence the speed of sound in liquid is given by the relation,

$$v = \sqrt{\frac{K}{\rho}}$$

Speed of Sound in Air or Gas

Newton's formula

the speed of sound in air is given by,

$$v = \sqrt{\frac{K}{\rho}} \quad \text{..(i)}$$

Where K is the bulk modulus of elasticity and ρ is the density of the medium. Newton assumed that when sound or longitudinal wave travels through air or gas medium, the compression and rarefactions are formed so slowly that heat which is produced in compression region is readily lost to the surrounding but at the same time the rarefaction region is cooled which gains the heat from the surrounding. So the temperature of the medium remains the same. Hence according to Newton, the condition is isothermal. He also proved that in this condition, the bulk modulus of gas is equal to the pressure of the gas.

Let V and P be the volume and pressure of a region of the medium where compression region is going to be formed. (P + dP) and (V – dV) are the pressure and volume of that region when compression is formed at constant temperature.

$$\therefore \quad K_{isothermal} = \frac{\text{stress}}{\text{strain}} = \frac{dP}{\frac{-dV}{V}} = -\frac{VdP}{dV} \quad \text{...............(ii)}$$

At constant temperature, according to Boyle's law

$PV = \text{constant}$

Differentiating both sides, we get

$PdV + V\,dP = 0$

or, $$P = -\frac{VdP}{dV}$$

$\therefore$ $P = K_{isothermal}$

Hence, $$v = \sqrt{\frac{K_{isothermal}}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \text{.........................(iii)}$$

At NTP , Density of air (ρ) = 1.293 kg/m^3

Atmospheric Pressure (P) = 760 mm of Hg = 1.013×10^5 N m^{-2}

$$\therefore \quad v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280 \text{ m s}^{-1}$$

But this value is less than its experimental value which is about 332 m/sec. This large difference between the theoretical and experimental value of sound in air at STP cannot be due to experimental error. Later, Laplace explained satisfactory reason for this discrepancy.

Laplace's correction

According to Laplace, the compression and rarefaction are formed so rapidly that neither heat is transferred to the surroundings nor heat is absorbed from the surrounding in order to obtain equilibrium for short interval of time. So according to him, the propagation of sound wave takes place under adiabatic condition. We have the gas equation for an adiabatic process as

$$PV^{\gamma} = \text{constant}$$

Where P, V and γ are the pressure of the region which is going to be compressed, its volume and the ratio of specific heat capacity at constant pressure to the specific heat capacity at constant volume of air. Differentiating this equation on both sides, we get

$$d(PV^{\gamma}) = d\,(\text{constant})$$

$$\text{or,}\quad \gamma PV^{\gamma-1}.dV + V^{\gamma}dP = 0$$

$$\text{or,}\quad \gamma P = -\frac{V^{\gamma}dP}{V^{\gamma-1}dV}$$

$$\text{or,}\quad \gamma P = \frac{-dP}{\frac{dV}{V}} = -\frac{VdP}{dV} \qquad = K$$

$$\therefore \quad K_{adiabatic} = \gamma P$$

Hence speed of sound in air is given by,

$$v = \sqrt{\frac{K_{adiabatic}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{..........................(iv)}$$

At NTP , Density of air (ρ) = 1.293 kg/m^3

Atmospheric Pressure (P) = 760 mm of Hg = 1.013×10^5 N m^{-2}

γ for air = 1.4

$$\therefore \quad v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.293}} = 331.5 \text{ m s}^{-1}$$

This value is very close to the experimental value. Hence Eq. (iv), derived by Laplace is used to determine the speed of sound in air.

Factors Affecting the Speed of Sound in Gases or Air

i. Effect of temperature

If M is the molar mass of gas and its volume is V, T is its temperature in Kelvin scale, then

$$PV = RT$$

Now, we know the velocity of sound in gas is, $v = \sqrt{\frac{\gamma P}{\rho}}$

Since $\rho = \frac{M}{V}$, $\quad v = \sqrt{\frac{\gamma PV}{M}}$

Again, $PV = RT$

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

For a given mass of air, $\frac{\gamma R}{M}$ is constant

$$\therefore \quad v \propto \sqrt{T \text{ (in K)}}$$

Hence the velocity of sound is directly proportional to the square root of temperature in Kelvin scale.

ii. **Effect of pressure**

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

but $\rho = \frac{M}{V}$ where M is the molar mass and V is its volume.

$$\therefore \quad v = \sqrt{\frac{\gamma PV}{M}}$$

we have PV = RT

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

At constant temperature $\frac{\gamma RT}{M}$ is constant for a given gas. *Hence the speed of sound is independent of atmospheric pressure at constant temperature.*

iii. **Effect of density of gas**

Let us consider two gases having different densities ρ_1 and ρ_2 respectively at the same pressure. If γ is same for both gases, the velocity of sound in these gases are given by

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

Dividing these two equations, we get $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

$$\therefore \quad v \propto \frac{1}{\sqrt{\rho}}$$

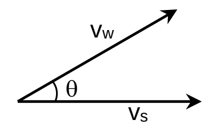
This shows that at constant pressure, the speed of sound in air is inversely proportional to the square root of density of the gas.

iv. **Effect of humidity**

The density of water vapour is smaller than that of dry air,and the presence of moisture in air reduces the density of air in the atmosphere.so, $\rho_{moist} < \rho_{dry}$ and as the velocity of sound in the gas is inversely proportional to the square root of its density, the velocity of sound in the moist air is greater than in the dry air. Greater the humidity of air,higher is the velocity of sound.Thus, sound travel faster in the air after rain than in the dry day.

v. Effect of wind

Let v_w and v_s are the velocity of wind and the velocity of sound in the given medium, θ is the angle between these two velocities. Then the resultant velocity of sound in the given direction is given by



$$v = v_s + v_w \cos\theta.$$

This shows that the velocity of sound in the given direction increases when θ is acute, decreases when θ is obtuse .

NUMERICAL PROBLEMS

1. A source of sound of frequency 550Hz emits waves of wavelength 60 cm in air at 20°C. What would be the wavelength of sound from the source in air at 0°C?

Ans: 0.58 m

2. What is the difference between the speed of longitudinal waves in air at 27°C and at – 13°C? What is the speed of 0°C?
Given, Density air at NTP (ρ) = 1.293 kg/m^3; Pressure (p) = 1.01×10^5 N/m^2

Ans: 23.96m/sec. 331.1 m/sec

3. At what temperature, the velocity of sound in air is increased by 50% to that at 27°C?

Ans: 675 K

4. Ini a resonance air column apparatus, the first and second reasonance position were observed at 18 cm and 56 cm respectively. The frequencuy of tuning fork used was 480 Hz. Calculate the velocity of sound in air and end correction of the tube.

Ans: 364.8 m/sec and 0.01 m

5. Calculate the bulk modulus of a liquid in which longitudinal waves the frequency of 250 Hz have the wavelength of 8 m if the density of liquid is 900 kg m^{-3}.

Ans:$3.6x10^9 N/m^2$

6. A source of sound produces a note of 512 Hz in air at 17°C with wavelength 66.5 cm. Find the ratio of molar heat capacities at constant pressure to constant volume at NTP. Densities of air and mercury at NTP are 1.293 kg/m^3 and 13600 kg/m^3 respectively.

Ans: 1.4

7. A Source of sound of frequency 512 Hz emits waves of wavelength 64.5 cm in air at 20°C. What would be the velocity of sound at 0°C?

Ans: 318.77m/sec

8. In a stormy day a boy observes a lightning flash which is followed by a thunder 3 sec later. How would you estimate the distance of the lightning strike from the boy. (given velocity of sound on that day = 332m/s, velocity of light c = 3 x 10^8m/s)

Ans: 996 m from boy

9. When a detonator is exploded on a railway line, an observer standing on the rail 2 km away hears two sounds. What is the time interval between them? (Young's modulus of steel = 2 × $10^{11}Nm^{-2}$, density of steel = 8 × 10^3 kg m^{-3}, density of air = 1.4 kgm^{-3}, γ- for air = 1.4, atmospheric pressure = 10^5Nm^{-2})

Ans: 5.92 Sec

10. A source of sound of frequency 512 Hz emits waves of wavelength 670 mm in air at 20°C. What is the velocity of sound in air at this temperature? What would be the wavelength of sound from the source in air at 0°C?

Ans: 343.04ms^{-1}, 0.65m

11. A man standing at one end of a closed corridor 57 m long blow a short blast on a whistle. He found that the time from the blast to the sixth echo was 2 seconds. If the temperature was 17°C, what was the velocity of sound at 0°C?

Ans: 331.8ms^{-1}

12. The interval between the flash of lighting and the sound of thunder is 2 seconds, when temperature is 10°C. How far is the storm if the velocity of sound in air at 0°C is 330ms^{-1}?

Ans: 672 m

Multiple choice question

1.If the speed of sound at 0^0C is v_0, the velocity at 273^0 C will be

(a) $2v_0$ (b) $0.5\ v_0$
(c) $1.4\ v_0$ (d) $0.7\ v_0$

2. **Keeping temperature constant, if the pressure is increase; the speed of sound in a gas**

(a) increases (b) decreases.
(c) remains unaffected. (d) cannot be predicted

3. **The speed of sound in a medium depends on**

(a) its elastic property but not one the inertia property.
(b) its inertia property but not on the elastic property.
(c) its elastic as well as inertia property.
(d) neither its elastic property nor the inertia property.

4. **Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ration of the speed of sound in gas 1 to that in gas 2 is give by**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

5. **The ration of velocity of sound in hydrogen gas (γ = 7/5) to helium gas (γ = 5/3) at some temperature is**

(a) $\sqrt{21}/5$ (b) $\sqrt{42}/5$
(c) $\sqrt{7}/5$ (d) $\sqrt{34}/5$

6. **The Laplace correction was necessary to Newton's calculation of velocity of sound, because propagation of sound in a medium is**

(a) Isothermal process (b) Adiabatic process
(c) Isobaric process (d) Isochoric process

Short Answer Questions

- **Velocity of sound in solids is more than that in liquids, why?**
- **The velocity of sound in solid is generally greater than that in gas at STP. Why? Give reason.**
- **When sound waves travel through a medium, does the temperature at various points remain constant? Explain.**
- **Why does sound travel faster in metals than in air?**
- **The speed of sound in humid air is more than that in dry air, why?**
- **Although the density of solid is high, the velocity of sound is greater in solid, explain.**
- **Is it possible that the velocity of sound is greater in solid than that in a gas at STP? Justify your answer.**
- **Velocity of sound increases on a cloudy day, why?**
- **Sound at a distance can be heard distinctly at night than in the day time, why?**
- **Explain why the velocity of sound in solids is greater than that in gases, though the densities of solids are greater than that of gases.**
- **Why are sounds heard better on a wet day than on a dry day?**
- **Is velocity of sound more in damp air or in dry air? Explain.**

THE END.