

RATE OF HEAT FLOW

Conduction

The process of transfer of heat within a medium from one point to another point without the actual movement of the molecules is conduction. During conduction, heat is carried by means of collision between vibrating molecules. This phenomenon is strongly applicable in solid only. For example, If one end of a metallic rod is heated, its next end also receives heat by conduction.

Quantitative Explanation of heat flow

The measure of ability of solid to conduct heat through it is called thermal conductivity.

Let us Consider a cubical metallic slab having, each side of length 'x'. ABCD and EFGH are two faces of the slab along x-axis. If the face ABCD is maintained at higher mean temperature $\theta_1^\circ\text{C}$ and the face EFGH is maintained at lower mean temperature $\theta_2^\circ\text{C}$ then the quantity of heat flowing through the section of the slab depends upon following factors.

1. The flow of heat is directly proportional to the cross sectional area.
i.e. $Q \propto A$ (i)
2. The flow of heat is directly proportional to the difference in temperature in between the faces,
i.e. $Q \propto (\theta_1 - \theta_2)$ (ii)
3. The flow of heat is directly proportional to the time interval for which heat is flown.
i.e. $Q \propto t$ (iii)
4. The flow of heat is inversely proportional to the separation between those two faces,
i.e. $Q \propto \frac{1}{x}$ (iv)

Combining equation (i), (ii), (iii) and (iv), we get,

$$Q \propto \frac{A(\theta_1 - \theta_2) t}{x}$$

$$\text{Or, } Q = \frac{KA (\theta_1 - \theta_2) t}{x}$$

Where the proportionality constant k is known as coefficient of thermal conductivity.

$$\text{Or, } K = \frac{Qx}{A (\theta_1 - \theta_2) t}$$

But if $x = 1\text{m}$, $A = 1\text{m}^2$, $(\theta_1 - \theta_2) = 1\text{K}$ and $t = 1\text{sec}$. Then,

$$K = Q.$$

Thus, coefficient of thermal conductivity is defined as the flow of heat per second in between two faces each having unit cross sectional area separated by unit distance apart and are maintained at unit degree change in temperature.

$$\begin{aligned} \text{As, } K &= \frac{Qx}{A (\theta_1 - \theta_2) t} \\ &= \frac{\text{Jm}}{\text{m}^2 \text{ks}} \\ &= \frac{\text{J}}{\text{mks}} \end{aligned}$$

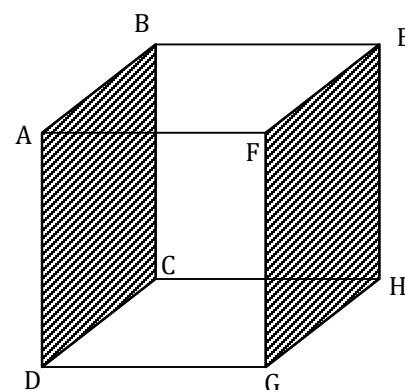


Fig. Thermal conductivity

$$= \text{W m}^{-1} \text{K}^{-1} \text{ (watt per meter per Kelvin)}$$

So, SI unit of coefficient of thermal conductivity is $\text{Wm}^{-1}\text{K}^{-1}$. And its dimension is $[\text{MLT}^{-3} \text{K}^{-1}]$

Determination of coefficient of Thermal conductivity of Good conductor by Searle's Method

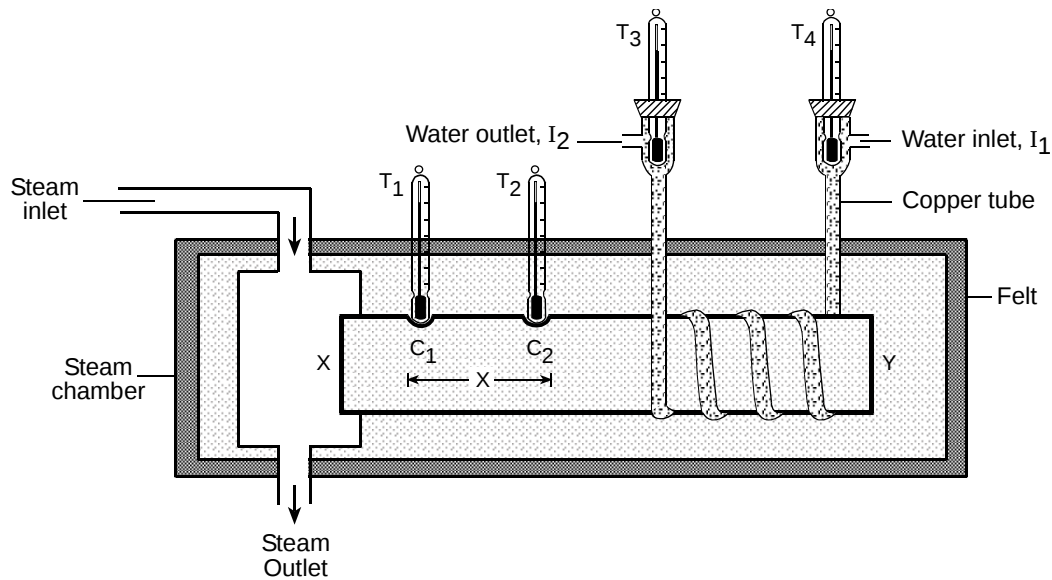


Fig. Determination of coefficient of thermal conductivity by Searle' s method

Let us Consider a metallic rod XY whose thermal conductivity is to be determined is lagged by using insulating material like felt to avoid the loss of heat due to radiation. The end X of the conductor is heated by passing steam continuously as shown in Fig: two small cavities C_1 and C_2 separated by distance 'x' are made then to ensure the good thermal contact between bulbs of thermometer and conductor, the cavities is filled with mercury. A copper pipe through which cold water can be run is wound near the end Y of the rod. The running water through the pipe can absorb heat from the rod and run out. The water coming out can be collected in a beaker to measure the mass of water.

The thermometer T_3 and T_4 measure the temperature of water coming out from the copper pipe and water passing inside the pipe respectively. When the steam is passed inside the steam chamber, initially the end X of the rod receives heat and then flow of the heat towards the cold end of the rod 'Y' takes place. At first the thermometer shows the increase in the reading. But the mass of water circulating in definite interval of time is maintained in such a way that it just can absorb the excess heat flowing on each section of the rod. After sometime all the four thermometers show constant readings and the state is known as steady state. The mass of water for the definite time is collected in a beaker and its mass is measured.

If 'A' be the cross sectional area of the rod, θ_1 , θ_2 , θ_3 and θ_4 be the temperature read by the thermometers T_1 , T_2 , T_3 and T_4 respectively during steady state condition and 't' be the time interval then heat flowing in between the cavities C_1 and C_2 be written as,

$$Q_1 = \frac{KA (\theta_1 - \theta_2) t}{x} \dots\dots\dots (i)$$

Where 'K' is the coefficient of thermal conductivity of the material

If 'm' and 's' be the mass and specific heat capacity of the water collected in the beaker during steady state. Then the heat absorbed by the water can be written as,

$$Q_2 = ms\Delta\theta$$

$$Q_2 = ms (\theta_3 - \theta_4) \dots\dots\dots (iv)$$

But for steady state, heat flowing on each section of the rod is equal. i.e.

$$Q_1 = Q_2$$

$$\text{or, } \frac{KA (\theta_1 - \theta_2) t}{x} = ms (\theta_3 - \theta_4)$$

$$\text{or, } \boxed{K = \frac{ms (\theta_3 - \theta_4) x}{A (\theta_1 - \theta_2) t}}$$

By knowing all the values of the parameters in RHS of the expression, we can easily find the coefficient of thermal conductivity of the metallic rod. In this way by using Searle's method we can find coefficient of thermal conductivity of good conductor.

Convection

It is the process of transfer of heat within a medium from one point to another point due to the actual movement of the molecules. During convection, the molecules near to the source of heat receive heat and its density decreases. With decreased density, the molecules become lighter and move upward. To fulfill the gap, the cold molecule nearer to it reaches there. Due to this continuous process, all molecules are heated. There are two convection processes .

1. Free convection:

When a hot body is in contact with air under normal condition, it receives heat from the body by a process called free or natural convection. Example, the air of the room removes heat of room by free convection.

2. Forced convection:

If the material is forced to move by a blower or pump , the process is called forced convection.e.g – ventilations system in room,human circulatory system etc.

Experimentally it is found that the rate of transfer of heat $\left(\frac{Q}{t}\right)$ is

- a. Directly proportional to the surface area of the fluid exposed to surrounding (A)

$$\text{i.e. } \frac{Q}{t} \propto A \dots\dots\dots (i)$$

- b. Directly proportional to the temperature difference between two parts of the fluid ($\Delta\theta$)

$$\text{i.e. } \frac{Q}{t} \propto \Delta\theta \dots\dots\dots (ii)$$

From equation (i) and (ii)

$$\frac{Q}{t} \propto A \Delta \theta$$

or, $\frac{Q}{t} = \beta A \Delta \theta$ (iii)

Where β is known as convection coefficient

Radiation

It is the process of transfer of heat from one place to another in a straight line with the speed of light without heating the intervening medium. During radiation, medium is not required but heat is transferred.

Reflection, Absorption and Transmission coefficient of heat radiation

When a heat radiation falls upon surface of any object, part of it is reflected, part of it is absorbed and part of it is transmitted out.

Consider a heat radiation of energy Q falls on surface of any object. 'R' amount of heat reflects, 'A' amount of heat is absorbed and T amount of heat is transmitted out. So, mathematically we can write.

$$Q = R + A + T \text{ (i)}$$

Dividing the equation (i) by Q we get,

$$1 = \frac{R}{Q} + \frac{A}{Q} + \frac{T}{Q}$$

Or, $1 = r + a + t \text{ (ii)}$

Where r is known as reflectance, a is absorptance and t is known as transmittance.

1. Reflectance (Coefficient of reflection)

It is defined as the ratio of the amount of heat radiation reflected by the body to the amount of heat radiation incident on it.

i.e. Reflectance (r) = $\frac{R}{Q}$

2. Absorptance (Coefficient of Absorption)

It is defined as ratio of the amount of heat radiation absorbed by the body to the heat radiation incident on it.

i.e. Absorptance (a) = $\frac{A}{Q}$.

3. Transmittance (Coefficient of Transmittance)

It is defined as the ratio of the amount of heat radiation transmitted by the body to the heat radiation incident on it.

i.e. Transmittance (t) = $\frac{T}{Q}$

If $r = 0$ and $t = 0$ then equation (ii) can be written as,

$$a = 1$$

This means all the radiations are absorbed and the body is known as perfectly black body.

Black body:

The body which can absorb most of the radiations fallen upon it is known as black body and the body which can absorb all the radiations fallen upon it is known as perfectly black body. When a black body is heated, at suitable temperature, it can emit the radiation captured by it. That's why black body is said to be a good absorber as well as good emitter. Black colored clothes are preferable in winter because it absorbs the surrounding radiation and increases the temperature of our body.

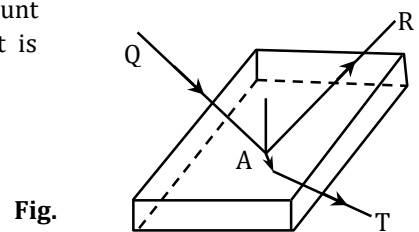


Fig.

Ferry's black body:

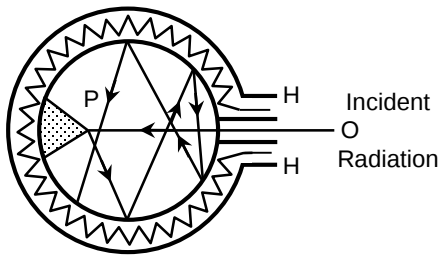


Fig. Ferry's black body.

The simplest and most commonly used black body was designed by Ferry. Fig. shows Ferry's black body. It consists of a hollow double walled metallic sphere having small opening O on one side and conical projection P just opposite to it. The inner walls of the sphere are coated with lamp black. Any radiation entering the hole is reflected many times at the inner walls of the sphere. After a few reflections, almost entire radiation is absorbed. The conical projection P does not permit any direct reflection.

When this black body is heated, the radiation will come out from the hole. Therefore, the hole (O) now acts as a black body radiator. Since it comes from hole, it is sometimes called cavity radiation. It is important to note that only the hole (and not the walls) acts as a black body radiator.

Emissivity

The emissivity of a body is the ratio of the emissive power of the body to the emissive power of perfectly black body at the same temperature. It is denoted by e .

Emissivity, $e = \frac{\epsilon}{E}$, where E is emissive power of perfectly black body at the same temperature as that of the given body. Its value ranges from 0 to 1. Dark and rough surfaces have emissivity near to 1 while bright and shiny surface have near to zero.

Stefan- Boltzmann's law of Black body radiation

According to this law, "The energy radiated per unit area per second by a perfectly black body is directly proportional to the fourth power of absolute temperature of the body".

i.e. Energy radiated per unit area per second (E) $\propto T^4$ (i)

Where T is the absolute temperature of the body

or, $E = \sigma T^4$ (ii)

Where ' σ ' is Stefan's constant whose value is $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ in .

Therefore rate of heat energy radiated (P) = σAT^4 (iii)

Where A = area of perfect black body.

If the body is not perfectly black body then equation (iii) can be written as,

$P = e\sigma AT^4$ (iv) Where ' e ' is emissivity of body.

If a perfectly black body at temperature T is surrounded by another perfectly black body at temperature T_0 . Then this law can be written as,

$$E = \sigma (T^4 - T_0^4) \dots\dots\dots (v)$$

If none perfectly black body at temperature T is surrounded by another perfectly blackbody at temperature T_0 . Then this law can be written as

$$E = e\sigma (T^4 - T_0^4) \dots\dots\dots (vi)$$

Then, rate of heat energy radiated, $P = e\sigma A (T^4 - T_0^4)$

Solar Constant

It is defined as the amount of heat radiation received from the sun per unit area per second by a perfectly black body placed on the earth's surface, when the area is perpendicular to the directions of rays. It is denoted by 'S'.

Solar constant (S) = $\sigma T^4 \left(\frac{R}{r}\right)^2$, Where T = temperature of sun, R = Radius of sun, r = astronomical distance.

Mathematically, $S = 1.388 \times 10^3 \text{ watt/m}^2$
 $\approx 1400 \text{ watt/m}^2$

Estimation of surface Temperature of Sun

Let us consider the sun of radius 'R' and solar constant 'S' is to be perfect black body at absolute temperature 'T'. Let r be the distance of the earth from the sun as shown in figure.

Now,

According to Stefan's law,

The total energy radiated per unit area per second by the sun is,

$$E = \sigma T^4 \dots\dots\dots (i)$$

$\therefore$ Total energy radiated per unit time,

$$P = \sigma AT^4$$

$$= \sigma 4\pi R^2 T^4 \dots\dots\dots (ii) \text{ Where, A = surface area of sun}$$

Again,

The total energy received per second by the sphere is

$$P = 4\pi r^2 S \dots\dots\dots (iii)$$

Where, S = solar constant

From equation (ii) and (iii)

$$\sigma T^4 \cdot 4\pi R^2 = S \cdot 4\pi r^2$$

$$T = \left(\frac{r^2 S}{R^2 \sigma}\right)^{1/4}$$

Here,

Solar constant (S) = 1400 W/m^2

Radius of the sun (R) = $7 \times 10^8 \text{ m}$

Distance of the sun from the earth (r) = $1.5 \times 10^{11} \text{ m}$

Stefan's constant (σ) = $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

Then, $T = 5802.73\text{K} \approx 6000\text{K}$

Hence, the temperature of the sun is approximately 6000 K.

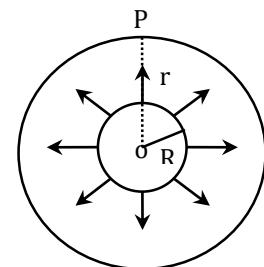


Fig. Heat radiated by the

Short Answer Questions

1. What is blackbody? How it is realized in practice?

Ans: The body which can absorb most of the radiations fallen upon it is known as blackbody. The body which can capture all the radiations fallen upon it is known as perfectly blackbody. However, perfectly black

body is hard to realize in practice in this nature. But in this universe, the black hole can be assumed to be perfectly black body. Ferry's black body is the one which is realized in practice.

2. Cooking utensils are blackened at the bottom and polished on the upper surface. Explain why?

Ans: Black surface is good absorber of radiation. If cooking utensils can absorb more heat from heating source, the food is cooked fast. If the bottom is blackened then it absorbs more radiation from heating source. The polished surface is good reflects of radiation. So if the upper surface is polished, the heat trying to escape from the upper surface are reflected back to the food and food is cooked fast. Due to this reason the bottom is blackened and upper surface is polished.

3. We feel metal door colder than the wooden door even when both of them are at same temperature. Why?

Ans: If heat is transferred quickly, the object is felt more cold or hot within short interval of time. Even though the metal door and wooden door are at same temperature but if we touch them due to greater thermal conductivity of metal, heat is transferred to the metal door from our body faster than wooden door. Due to this reason the metal door is felt colder than wooden door.

4. How can water be boiled in a paper cup?

Ans: Yes water can be boiled in a paper cup. The ignition temperature (The temperature at which paper burns) is very much greater than boiling point of water. When water is kept in a thin paper cup and is heated, the heat transfers to the water by the paper and due to convection it starts to boil before the paper ignites.

5. In winter, birds often swell their feathers, why?

Ans: The birds enclose a lot of air by spreading the wings. As the air is bad conductor of heat, it does not allow the internal heat of the bird to escape and hence they are protected from cold.

6. Why animal curl their bodies in severe cold?

Ans: Amount of heat radiated by a body is proportional to the surface area. The volume and then surface of a body are minimum when it is in spherical shape. Therefore, animals curl their body to decrease the surface area and in turn to decrease the amount of heat loss or to preserve the heat of the body.

7. Why are two thin blankets warmer than a single blanket of double the thickness?

Ans: When two thin blankets are used, there is a layer of air enclosed between two blankets and provides a medium of bad conductor of heat which reduces the rate of heat lost from body to surrounding therefore two blankets are warmer than a single blankets of double thickness.

8. Although aluminum is good conductor of heat, how can aluminum foil of shinny surface is used to keep food warmer for long time?

Ans: we know shinny surface is good reflector of radiation. If food is wrapped with aluminum foil of shinny surface, the heat emitting from food is reflected back towards the food itself. Its ratio of conduction of heat through the foil with reflection is too much small. So the food remains warm for longer time.

NUMERICALS PROBLEMS

- 1. A bar 0.20 m in length and of cross sectional area 2.5 cm^2 is ideally lagged. One end is maintained at 373 K while the other is maintained at 273 K by immersion in melting ice. Calculate the rate at which the ice melts owing to the flow of heat along the bar. (Thermal conductivity of the material of the bar = $4.0 \times 10^2 \text{ Wm}^{-2}\text{K}^{-1}$ and specific latent heat of fusion of ice = $3.4 \times 10^5 \text{ JKg}^{-1}$.)**

Solution:

Here, length of the bar (l) = 0.2 m
Cross section area of bar (A) = $2.5 \text{ cm}^2 = 2.5 \times 10^{-4} \text{ m}^2$
Temperature at one end (θ_1) = 373 K
Temperature at another end (θ_2) = 273 K
 $K = 4 \times 10^2 \text{ Wm}^{-1}\text{K}^{-1}$

$$L_{\text{ice}} = 3.4 \times 10^5 \text{ J/Kg}$$

Let m kg of ice gets melted

$\therefore$ Heat conducted through bar = Heat gained by ice

$$\text{or, } \frac{KA(\theta_1 - \theta_2)t}{l} = m L_{\text{ice}}$$

$$\frac{KA(\theta_1 - \theta_2)}{l} = \frac{m}{t} L_{\text{ice}}$$

$$\text{or, } \frac{4 \times 10^2 \times 2.5 \times 10^{-4} (373 - 273)}{0.2} = \frac{m}{t} \times 3.4 \times 10^5$$

$$\text{or, } \frac{10 \times 10^2 \times 10^{-4} \times 100}{0.2} = \frac{m}{t} \times 3.4 \times 10^5$$

$$\text{or, } \frac{10}{0.2} = \frac{m}{t} \times 3.4 \times 10^5$$

$$\therefore \frac{m}{t} = \frac{10}{0.2 \times 3.4 \times 10^5} = 1.47 \times 10^{-4} \text{ kg/s}$$

2. A slab of stone of area 0.36m^2 and thickness 10cm is exposed on the lower surface to steam at 100°C . A block of ice at 0°C rests on the upper surface of the slab. In one hour 4.8 kg of ice is melted. Calculate the thermal conductivity of stone. [Latent heat of fusion ice = $3.36 \times 10^5 \text{ J/kg}$]

Solution:

$$\text{Area of stone (A)} = 0.36\text{m}^2$$

$$\text{Thickness (x)} = 10\text{cm} = 10 \times 10^{-2}\text{m}$$

$$\text{Temperature at lower surface } (\theta_1) = 100^\circ\text{C}$$

$$\text{Temperature at upper surface } (\theta_2) = 0^\circ\text{C}$$

$$\text{Mass of ice melted } (m_i) = 4.8\text{kg}$$

$$\text{Time taken (t)} = 1\text{hr}$$

$$= 60 \times 60 = 3600\text{sec.}$$

$$\text{Latent heat of fusion of ice } (L_f) = 3.36 \times 10^5 \text{ J/kg}$$

Now,

$$\text{Heat transferred (Q)} = \frac{KA(\theta_1 - \theta_2)t}{x}$$

Again,

For melting ice,

$$m_i L_f = \frac{KA(\theta_1 - \theta_2)t}{x}$$

$$\begin{aligned} \text{or, } K &= m_i L_f \times \frac{x}{A(\theta_1 - \theta_2)t} \\ &= \frac{4.8 \times 3.36 \times 10^5 \times 10 \times 10^{-2}}{0.36 \times (100 - 0) \times 3600} \\ &= 1.244 \text{ Wm}^{-1}\text{K}^{-1}. \end{aligned}$$

3. What is the ratio of the energy per second radiated by the filament of a lamp at 2500K to that radiated at 2000K , assuming the filament is a black body radiator?

Solution

We know, Energy radiated per second (P) = σAT^4

For temperature (T_1) = 2500K and temperature (T_2) = 2000K , we have

$$\frac{P_1}{P_2} = \frac{\sigma AT_1^4}{\sigma AT_2^4}$$

$$\frac{P_1}{P_2} = \frac{T_1^4}{T_2^4}$$

$$\frac{P_1}{P_2} = \frac{2500^4}{2000^4}$$

$$\frac{P_1}{P_2} = 2.441$$

Hence the ratio of power radiated is 2.44:1

4. Estimate the rate of heat loss through a glass window of area 2m^2 and thickness of 3mm when the temperature of the room is 20°C and that of air outside is 5°C . Given Thermal conductivity of glass $= 1.2\text{Wm}^{-1}\text{K}^{-1}$.

Solution:

Here,

$$\text{Area of window (A)} = 2\text{m}^2$$

$$\text{Thickness (d)} = 3 \times 10^{-3}\text{m}$$

$$\text{Temperature inside } (\theta_1) = 20^\circ\text{C}$$

$$\text{Temperature outside } (\theta_2) = 5^\circ\text{C}$$

$$\text{Thermal conductivity of glass} = 1.2\text{Wm}^{-1}\text{K}^{-1}$$

$$\text{Rate of heat lost (Q/t)} = ?$$

Using the formula,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\frac{Q}{t} = \frac{1.2 \times 2(20 - 5)}{3 \times 10^{-3}}$$

$$\frac{Q}{t} = 12000 \text{ Watt}$$

5. Assuming that the thermal insulation provided by a woolen glove is equivalent to a layer of quiescent air 3mm thick, determine the heat loss per minute from a man's hand, surface area 200cm^2 on a winter day when the atmospheric air temperature is -3°C . The skin temperature is to be taken as 35°C and thermal conductivity of air as $24 \times 10^{-3}\text{Wm}^{-1}\text{K}^{-1}$.

Solution

$$\text{Thickness of layer (d)} = 3 \times 10^{-3}\text{m}$$

$$\text{Surface area (A)} = 200\text{cm}^2 = 200 \times 10^{-4}\text{m}^2$$

$$\text{Temperature of the body } (\theta_1) = 35^\circ\text{C}$$

$$\text{Atmospheric air temperature } (\theta_2) = -3^\circ\text{C}$$

$$\text{Thermal conductivity of air (K)} = 24 \times 10^{-3}\text{Wm}^{-1}\text{K}^{-1}$$

$$\text{Heat lost per minute } \left(\frac{Q}{t}\right) = ?$$

Using the formula,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\frac{Q}{t} = \frac{24 \times 10^{-3} \times 200 \times 10^{-4}(35 + 3)}{3 \times 10^{-3}}$$

$$\frac{Q}{t} = 6080000 \frac{\text{joule}}{\text{s}}$$

$$\frac{Q}{t} = 3.65 \times 10^8 \text{ joule per minute}$$

6. A man, the surface area of whose skin is 2m^2 is sitting in a room where the air temperature is 20°C . If his skin temperature is 37°C , find the rate at which his body loses heat. The emissivity of his skin is 0.97.

Solution: Surface area of man skin (A) = 2m^2

$$\text{Skin temperature (T)} = 37^\circ\text{C} = 310\text{K}$$

$$\text{Air temperature (T}_0\text{)} = 20^\circ\text{C} = 293\text{K}$$

$$\text{Emissivity of skin (e)} = 0.97$$

$$\text{Rate of heat lost (P)} = ?$$

Using the formula, $P = e\sigma A(T^4 - T_0^4)$

$$P = 0.97 \times 5.67 \times 10^{-8} \times 2 \times [(310)^4 - (293)^4]$$

$$P = 92.2 \text{ W}$$

7. The sun is a black body of surface temperature about 6000K. If the sun's radius is $7 \times 10^8\text{m}$, calculate the energy per second radiated from its surface. (Stefan's constant = $5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$)

Solution

$$\text{Temperature of sun (T)} = 6000\text{K}$$

$$\text{Radius of sun (R)} = 7 \times 10^8\text{m}$$

$$\text{Surface area of sun (A)} = 4\pi R^2 = 4\pi (7 \times 10^8)^2 = 6.157 \times 10^{18} \text{m}^2$$

$$\text{Stefan's constant } (\sigma) = 5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$$

Energy radiated per second by the sun (P) = ?

$$\text{Now, } P = \sigma AT^4 = 5.7 \times 10^{-8} \times 6.157 \times 10^{18} \times 6000^4 = 4.54 \times 10^{26} \text{ Watt}$$

8. Calculate the temperature of the sun from the following information; Sun's radius = $7.04 \times 10^5 \text{ km}$, distance from the sun = $14.72 \times 10^7 \text{ km}$, Solar constant = 1400Wm^{-2} and Stefan's constant = $5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$

Solution

Here,

$$\text{Radius of sun (R)} = 7.04 \times 10^5 \text{ km} = 7.04 \times 10^8 \text{m}$$

$$\text{Distance of sun from earth (r)} = 14.72 \times 10^7 \text{ km} = 14.72 \times 10^{10} \text{m}$$

$$\text{Solar constant (S)} = 1400 \text{Wm}^{-2}$$

$$\text{Stefan's constant } (\sigma) = 5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$$

Let T be the temperature of the sun then

$$\text{The power radiated by the sun (P)} = \sigma AT^4 = 5.7 \times 10^{-8} \times 4\pi R^2 \times T^4 \dots\dots\dots (i)$$

The total energy received per second by the sphere is

$$P = 4\pi r^2.S \dots\dots\dots (ii)$$

From equation (i) and (ii) $5.7 \times 10^{-8} \times 4\pi R^2 \times T^4 = 4\pi r^2.S$

$$T^4 = \frac{4\pi r^2 S}{5.7 \times 10^{-8} \times 4\pi R^2}$$

$$= \frac{4\pi (14.72 \times 10^{10})^2 \times 1400}{5.7 \times 10^{-8} \times 4\pi (7.04 \times 10^8)^2}$$

$$T = 5451^\circ\text{C}$$

NUMERICALS

1. A bar 0.2m in length and 2.5 cm^2 in cross section is ideally lagged. One end is maintained at 100° and the other end is maintained at 0° C by immersing in melting ice. Calculate the mass of ice melt in one hour: Thermal conductivity of the material of the bar is $4 \times 10^{-2} \text{ Wm}^{-1} \text{ K}^{-1}$.

Ans: $5.36 \times 10^{-5} \text{ kg}$

2. The Sun is a black body of surface temperature about 6000K. If Sun's radius is $7 \times 10^8 \text{m}$, calculate the energy per second radiated from its surface. The earth is about $1.5 \times 10^{11} \text{m}$ from the Sun. Assuming all the radiation from the Sun falls on the surface of sphere of this radius, estimate the energy per second per meter² received by the earth.
Ans: $4.55 \times 10^{26} \text{ W}$, 1609 Wm^{-2}
3. A sphere of radius 2.00 cm with a black surface is cooled and then suspended in a large evacuated enclosure with black walls maintained at 27°C. If the rate of change of thermal energy of sphere is 1.85 J s^{-1} received by the earth.
Ans: $5.66 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$
4. Estimate the power loss through unit area from a perfectly black body at 327°C to the surrounding environment at 27°C.[4]
Ans: 6889.05 W
5. A spherical blackbody of radius 5cm has its temperature 127°C and its emissivity is 0.6. Calculate its radiant power.
Ans: 27.34 W
6. Estimate the radiant power loss from a human body at a temperature 38.5°C to the environment at 0°C if the surface area of the body is 1.5 m² and its emissivity is 0.6
Ans: 197 W
7. A pot with a steel bottom 8.5 mm thick rest on a hot stove. The area of the bottom of the pot is 0.15m². The water inside the pot is at 100°C and 390 gm of water is evaporated every 3 minute. Find the temperature of lower surface of the pot which is in contact with the stove. [$k = 50.2 \text{ W/mK}$, $L_v = 2256 \times 10^3 \text{ J/Kg}$]
Ans: 105.52°C
8. A rod 1.3m long consists of a 0.8 m length of aluminium joined end to end to a 0.5 m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece if maintained at 20°C. No heat is lost through the sides of the rod. At a steady state, what is the temperature at the point where the two metals are joined? [$K_{al} = 205 \text{ W/mK}$, $K_b = 110 \text{ w/mK}$]
Ans: 90°C
9. A rod 1.3 m long consists of a 0.8 m length of aluminium joined end to end to a 0.5m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece is maintained at 20°C. No heat is lost through the side of the rod. At steady state what is the temperature of the point when two metals are joined.
Ans: 90°C
10. The element of an electric fire with an output of 1.5 Kw is a cylinder of 0.3 m long and 0.04 m in radius. Calculate its temperature if it behaves as a black body.
Ans: 775.64k
11. A bar 0.2 m in length and of cross-sectional area $2.5 \times 10^{-4} \text{m}^2$ is ideally lagged. One end is maintained at 373 K while the other is maintained at 273 K by immersing in melting ice Calculate the rate at which the ice melts owing to the flow of heat along the bar.
Ans: $1.5 \times 10^{-4} \text{ kg/sec}$
12. A slab of stone of area 0.36 m² and thickness 10 cm is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on the upper surface of the slab. In one hour, 4.8 kg of ice is melted. Calculate the thermal conductivity of stone.
Ans: 1.24 watt m⁻¹k⁻¹
13. A metal rod of length 20cm and cross sectional area 3.14 cm² is covered with non-

conducting substance. One of its end is maintained at 100°C , while the other end is put in ice at 0°C . It is found that 25 gram of ice melts in 5 minutes. Calculate the thermal conductivity of the metal.

Ans: 178.3 watt/mk

14. Estimate the rate of heat loss through a glass window of area 2m^2 and thickness 4mm when the temperature of the room is 300K and temperature outside is 5°C .

Ans: Given, $K = 1.2 \text{ Wm}^{-1}\text{K}^{-1}$, 13200 Watt

15. The sun is a black body of surface temperature about 6000 K. If the sun's radius is $7 \times 10^8\text{m}$, calculate the energy per second radiated from its surface. (Stefan's constant = $5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$)

Ans: $4.55 \times 10^{26} \text{ W}$

16. An ice box is made of wood 1.75 cm. thick lined inside with cork 2 cm. thick. If the temperature of inner surface of the cork is steady at 0°C and that of the outer surface of the wood is steady at 12°C ; what is the temperature of the interface? The thermal conductivity of wood is five times that of cork.

Ans: 10.23°C

17. A man, the surface area of whose skin is 2m^2 is sitting in a room where the air temperature is 20°C . If his skin temperature is 37°C , find the rate at which his body loses heat. The emissivity of his skin is 0.97.

Ans: 205.2W

18. Assuming that the thermal insulation provided by a woolen glove is equivalent to a layer of quiescent air 3 mm thick, determine the heat loss per minute from a man's hand, surface area 200 cm^2 on a winter day when the atmospheric air temperature is -3°C . The skin temperature is to be taken as 35°C and thermal conductivity of air as $24 \times 10^{-3} \text{ Wm}^{-1}\text{K}^{-1}$.

Ans: 364.8J

19. Estimate the rate of heat loss through a glass window of area 2m^2 and thickness 3 mm when the temperature of the room is 20°C and that of air outside is 5°C .

Ans: 12000W

20. What is the ratio of the energy per second radiated by the filament of a lamp at 2500 K to that radiated at 2000 K, assuming the filament is a black body radiator?

Ans: 2.44:1

21. Estimate the rate at which ice would melt in a wooden box 2.5 cm. thick of inside measurement $100 \text{ cm} \times 60 \text{ cm} \times 40 \text{ cm}$. assuming that the external temperature is 35°C and thermal conductivity of wood is $0.168 \text{ Wm}^{-1}\text{K}^{-1}$.

Ans: $1.6 \times 10^{-3} \text{ kg/sec}$.