Radioactivity

The spontaneous emission of powerful radiation from the nuclei of the heavier element is called Radioactivity. The unstable nuclei are suitable for the radioactivity.

Cause of the Radioactivity

The nucleons in the nucleus are subjected to attractive nuclear force and coulomb repulsive force between the protons. Only the nuclei having a certain combination of protons and neutrons are stable. If the combination of protons and neutrons in a nucleus is such that the repulsive coulomb force between the protons is greater than the attractive nuclear force, the nucleus is unstable. As a result, the nucleus spontaneously disintegrates by emitting radiations in the form of α, β and γ-radiations.

Radioactive radiations

The radiations emitted by unstable radioactive nucleus are called radioactive radiations. they are named as the **α-particle, β -particle and γ- radiation**.

Type of radioactivity

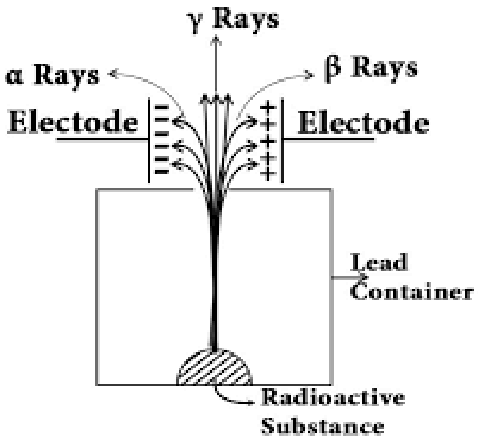
a. Natural radioactivity

The phenomenon of the spontaneous emission of highly penetrating radiations from heavy elements($A>206$) occurring in the nature is called natural radioactivity. This includes radio activities shown by natural radioactive source.

b. Artificial radioactivity

The phenomenon in which radioactivity can be induced by artificial means through nuclear transmutation is called artificial radioactivity. This includes radio activities shown by artificially made radioactive source.

Radiations emitted by Radioactive Elements



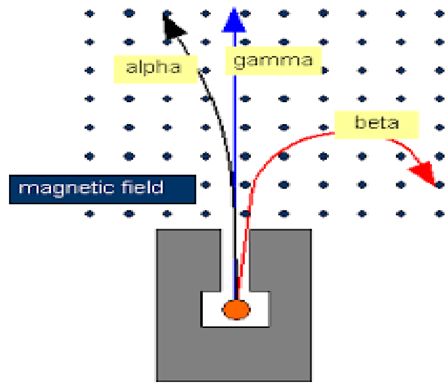


Fig: Three types of rays diffracted in electric field and in magnetic field

The radiations emitted by radioactive substance consists of α-particle, β-particle and γ radiation. To demonstrate the existence of three types of Particles in a radioactive radiation a sample of radioactive substance like Uranium is taken in a hole of the lead block as shown in figure. The radiations coming from the source is subjected to the electric field applied between two parallel plates containing positive and negative terminal. Then following observations are made:

i) The radiation which bends towards the negatively charge plate are called α**-particle.**

ii) The radiation which bends towards the positively charge plate are called **β -particle**.

iii) The radiation which goes straight consists of photons are called **γ-ray.**

Similarly , the existence of three types of Particles in a radioactive radiation a sample of radioactive can demonstrated in the presence of the magnetic field.

Properties of the α, β and γ -Particle

α-Particle

- It is the Nuclei of the helium atoms ($_2He^4$).

- They are Positively charge having charge +2e.
- The rest Mass is equal to 6.4×10^{-27}kg.
- The Energy of the **α** -particle emitted from radioactive substance is about 6Mev
- The Ionizing power is higher than β-Particle
- the Penetrating power is less than β-Particle

β-Particle

- It is the Nuclei having $_{-1}e^{0}$.
- They are Negatively charge having charge -e.
- The rest Mass is equal to 9.1×10^{-31}kg.
- They move with high velocity of the order of 10^{8} m/s.
- They are deflected in Electric Field(E) and Magnetic Field(B).
- Their energy range 2 to 3 Mev.
- The Ionizing power is $\frac{1}{100}$ th of α-particle.
- The Penetrating power is 100 times α-particle.

γ -Radiation

- They are electromagnetic radiation of the short wavelength.
- They are chargeless.
- They move with the velocity equal to that of light.
- They are not deflected in Electric Field(E) and Magnetic Field(B).
- The Ionizing power is 1/100 th of β-particle.
- The Penetrating power is 100 times β-particle.

Alpha Decay

When a nuclide emits α-particle, its mass number is reduced by four and atomic number by two. This process is known as Alpha Decay.

$$_{Z}X^{A} \quad \rightarrow \quad _{Z-2}Y^{A-4} \quad + \quad 2He^{4}$$

(Parent nucleus) (Daughter nucleus) (outgoing particle)

Example: $_{92}U^{235} \rightarrow {_{90}Th^{231}} + 2He^4$

Beta Decay

When a nuclide emits beta particle, its mass number remains same but atomic number increased by 1. This process is known as the Beta Decay.

$$_ZX^A \rightarrow {_{Z+1}Y^A} + {_{-1}e^0}$$

(Parent nucleus) (Daughter nucleus) (outgoing particle)

Example: $_6C^{14} \rightarrow {_7N^{14}} + {_{-1}e^0}$

Gamma Decay

When a nuclide emits gamma radiation, neither mass number nor atomic number change. This process is known as the Gamma Decay.

$$({_ZX^A})^* \rightarrow {_ZX^A} + \gamma$$

The γ-radiation is emitted by the excited nucleus.

Law of the Radioactive Disintegration

The breaking of the nucleus is known as radioactive disintegration. The law of the radioactive disintegration are as follows:

a) Radioactive decay is a **random** and **spontaneous process**.it cannot influenced by external conditions such as pressure, temperature, electric field etc.

b) In any radioactive decay either an α-particle orβ-particle is emitted by atom. Emission of the both is impossible at a time.

C)Decay law

The rate of disintegration of radioactive substance is directly proportional to the number of atoms present at that instant.

Decay Equation

Let N_o be the number of atoms present in the radioactive sample at t=0 and N be the number of the atoms left after time t. according to Decay law,

$$\frac{dN}{dt} \alpha N$$

$$\text{Or } \frac{dN}{dt} = -\lambda No$$

Where λ is a constant of proportionality called as disintegration constant or decay constant. (-)ve sign indicates that N decreases as time increases.

Now $\frac{dN}{N} = -\lambda dt$………..(1)

Integrating equation (1) both side

$$\int_{No}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt$$

or $[\log e N]_{No}^{N} = -\lambda\ [t]_{0}^{t}$

or $\log e\, N - \log_e N_o = -\lambda t$

or $\text{Log}_e \frac{N}{N_o} = -\lambda t$

or $\frac{N}{N_o} = e^{-\lambda t}$

$$\mathbf{N = N_o\, e^{-\lambda t}}$$

This equation is known as the decay equation. Its shows that number of active nuclei in a radioactive sample decreases exponentially with time as show in figure.

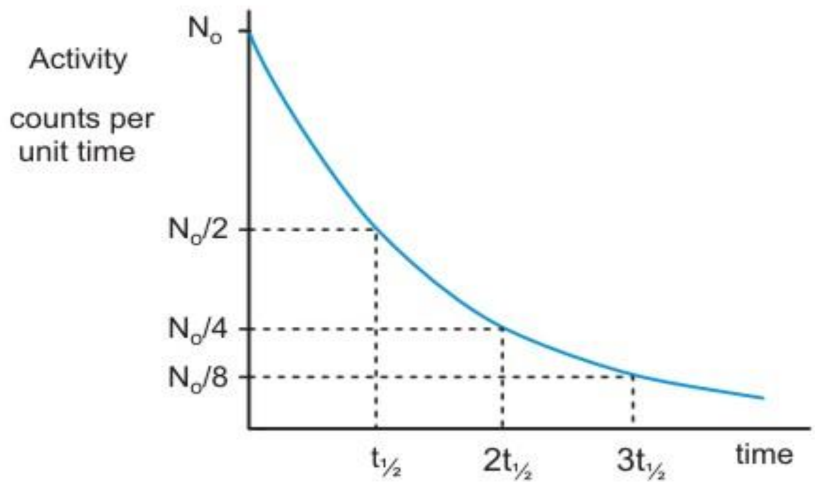


Fig: Radioactive Decay with time

Decay constant(λ)

From the law of radioactive disintegration, we have,

$$\frac{dN}{dt}=-\lambda N$$

$$\text{or} \quad \lambda = \frac{-\frac{dN}{dt}}{N}$$

Hence Decay constant is defined as the rate of decay per unit number of atom present at that time.

If we put $\lambda=1/t$ in decay equation $N=N_o e^{-\lambda t}$ we get

$N=N_o e^{-1}=\frac{N_o}{e}=\frac{N_o}{2.718}=0.37\ N_o=37\%$ of N_o

Hence, Decay constant may also be defined as the reciprocal of the time during which the number of radioactive atoms of radioactive substance falls to 37% of its original value.

Half-life period

The time taken by a radioactive substance to disintegrate or decay half of its atoms is called half-life period. It is denoted by $T_{1/2}$.

Relation between half-life($T_{1/2}$) and decay constant (λ)

From the decay Equation, $N=N_o e^{-\lambda t}$

At half-life, $N=\frac{N_o}{2}$ and $t= T_{1/2}$ then from the above equation,

$$\frac{N_o}{2}= N_o e^{-\lambda T_{1/2}}$$

Or $\frac{1}{2}= e^{-\lambda T_{1/2}}$

Or $2= e^{\lambda T_{1/2}}$

Or $\log_e 2= \lambda T_{1/2}$

Or $T_{1/2} =\frac{\log_e 2}{\lambda}$

$$\mathbf{T_{1/2} =\frac{0.693}{\lambda}}$$

Above equation is the required relation between half-life($T_{1/2}$) and decay constant (λ).

<u>Average Life or Mean Life</u>

It is defined as the ratio of the sum of total life of an atoms to the total number of atoms in the element.it is denoted by T_{mean}

Mean Life (T_{mean})$=\frac{\text{sum of total Life of an atoms}}{\text{total number of atoms}}$

In another word, Mean Life of the radioactive substance is eqaul to the reciprocal of the Decay constant.

$T_{mean}=\frac{1}{\lambda}$

But $\lambda =\frac{0.693}{T_{1/2}}$ so

$T_{mean}=\frac{T_{1/2}}{0.693}$

$T_{mean} =1.443\ T_{1/2}$

Thus, the mean life of a radioactive substance is longer than its half-life.

Activity of Radioactive substance(R)

The rate of decay of a radioactive substance is called the activity of the substance.

$R = \frac{dN}{dt} = -\lambda N$

or $R = \lambda N$…… (1) (taking magnitude)

So, $R \alpha N$.

If R_o is the activity of the substance at time t=0, then

$R_o = \lambda N_o$ ……. (2)

Dividing (1) by (2)

$\frac{R}{R_o} = \frac{N}{N_o} = \frac{N_o e^{-\lambda t}}{N_o} = e^{-\lambda t}$

$\mathbf{R = Ro\ e^{-\lambda t}}$

Number of the atoms left behind after n half lives

If N_o be the total number of atoms of a radioactive substance present at time t=0 then,

Number of atoms present after one half-life(N)= $\frac{N_o}{2}$

Number of atoms present after second half life(N)=$\frac{1}{2}\left(\frac{N_o}{2}\right) = \left(\frac{1}{2}\right)^2 N_o$

Number of atoms present after n-half-life$\mathbf{(N) = \left(\frac{1}{2}\right)^n N_o}$

Further, if the R be the activity after n half-life times and R_o be the initial activity, then, $\mathbf{R = \left(\frac{1}{2}\right)^n R_o}$

Units of Radioactivity

The activity of a substance is measured in terms of disintegration per second. Following are the unit of radioactivity.

i)**Curie (Ci):**

It is defined as the activity of a radioactive substances which gives 3.7×10^{10} disintegration per second.

i.e $1Ci = 3.7\times10^{10}$ disintegration/sec

ii)Rutherford(rd):

It is defined as the activity of a radioactive substances which gives 10^6 disintegration per second.

i.e $1rd = 10^6$ disintegration/sec

iii)Becquerel (Bq):

It is defined as the activity of a radioactive substances which gives 1 disintegration per second.

i.e 1Bq= 1 disintegration/sec

Radio Isotopes

The isotopes of an element which are also radioactive substance are called radioactivate isotopes or simply Radio isotopes. For example:

i)C^{14}, C^{12}, C^{11} are the isotopes of carbon. As C^{14} and C^{11} emits the radioactive radiation, they are known as radio isotopes of carbon.

ii)Na^{23} and N^{24} are the radio-isotopes of sodium

Radio Carbon Dating

Radiocarbon dating is a method that provides objective age estimates for carbon-based materials that originated from living organisms. An age could be estimated by measuring the proportion of ${}_6C^{14}$ to ${}_6C^{12}$ in the living organisms.

When neutron by cosmic rays reacts with nitrogen in air to form $_6C^{14}$ as below:

$_7N^{14} + _0n^1 \rightarrow _6C^{14} + _1H^1$

The ratio of C^{14} to C^{12} in the atmosphere is about 10^{-12}. Both C^{14} and C^{12} combine with oxygen to form carbon dioxide that is absorbed by the living plants during the photosynthesis process. But C^{14} is a β-emitter and decay back into nitrogen

$$_6C^{14} \rightarrow _7N^{14} + _{-1}e^0$$

Hence the amount of the $_6C^{14}$ goes on decreasing. As the living plant performs photosynthesis, the decay of $_6C^{14}$ is compensated by a new supply from atmospheric air and a radioactive equilibrium is obtained where there is the fixed ratio of $_6C^{14}$ to $_6C^{12}$.

When plant dies, no more $_6C^{14}$ is taken and that inside the plant begins to decay at the known rate without replacement. $_6C^{14}$ has the half-life of 5730 years. By measuring activity of $_6C^{14}$, we can determine life of the object.

Suppose R_0 be the activity of the dead material at the time of death and R be the activity after t years, then

$$\mathbf{R = R_o\, e^{-\lambda t}}$$

Or $\mathbf{e^{-\lambda t}} = \frac{\mathbf{R}}{\mathbf{R_o}}$

Or $\mathbf{e^{\lambda t}} = \frac{\mathbf{Ro}}{\mathbf{R}}$

Or $\lambda t = \log_e \left(\frac{\mathbf{Ro}}{\mathbf{R}}\right)$

Or $t = \frac{1}{\lambda} \log_e\left(\frac{\mathbf{Ro}}{\mathbf{R}}\right)$

Since $\lambda = \frac{0.693}{T1/2}$ then $t = \frac{T1/2}{0.693} \log_e\left(\frac{\mathbf{Ro}}{\mathbf{R}}\right)$

Hence by measuring the activity R_0 and R of the its dead material, the age of that material can be determined.

Geiger Muller Tube (Counter)

The Geiger Muller Tube is the simplest device used for the detection of the nuclear radiations or ionizing particles. The experimental arrangement of the Geiger muller Tube is show in figure below

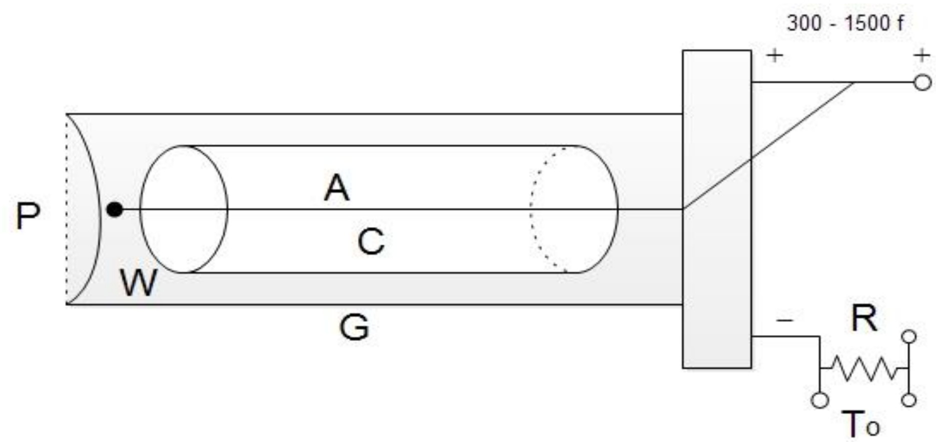


Fig: Geiger Muller tube

Construction:

It consists of a glass tube G coated with the metal inside it as the cathode C.A metal wire runs along the axis of the cylinder as anode A as shown in figure. The cylinder is filled with a gas such as argon mixed with halogen vapour at a low pressure about 10cm of Hg. The tube has very thin mica end window W shielded by a protective gauge P. The high-tension source (H.T) supplies a potential difference less than the ionizing potential of the gas through the resistance R

Working Principle:

A high energy ionizing particle like a β-particle or γ-radiation entering through the mica window W will cause one or more of the argon atoms to ionize. The electrons and ions of argon thus produced cause other argon atoms to ionize due to collision. The result of this one event is sudden, massive electrical discharge that causes a current pulse. The current through R produces a small voltage pulse. An electron pulse amplifier accepts the small pulse voltage and amplifies them. The amplified output is then applied to a counter. As each incoming particle produces a pulse, the number of incoming particles can be counted.

Formula for the Numerical Problem

1. $\boldsymbol{\frac{dN}{dt} = \lambda N}$ (taking magnitude only)
2. $\boldsymbol{N = N_o e^{-\lambda t}}$

Where N= number of the atoms left after time ‘t’ or number of undecayed atom

N_o =initial number of atoms

$N_o - N$ = number of decay atom

λ= Decay constant

3. $\boldsymbol{T_{1/2} = \frac{0.693}{\lambda}}$ Where $T_{1/2}$ =Half-life Period
4. $\boldsymbol{T_{mean} = \frac{1}{\lambda}}$
5. $\boldsymbol{R = R_o e^{-\lambda t}}$

R= activity of the atoms at time ‘t’

R_o =initial activity

Numerical Problem

1. If the half-life period of the radioactive substance is 2 days, after how many days will $\frac{1}{64}$th part of the substance be left behind?

Solution: Half-life($T_{1/2}$) =2 days

$N = \frac{1}{64}$ of $N_0 = \frac{1}{64} \times N_0 = \frac{N_0}{64}$

time(t)=?

we know that,

$N = N_o e^{-\lambda t}$

$\frac{N_0}{64} = N_o e^{-\lambda t}$

$\frac{1}{64} = e^{-\lambda t}$

or $64 = e^{\lambda t}$

or $\mathrm{Log}_e 64 = \lambda t$

or $\mathrm{Log}_e 64 = t \times \frac{\mathbf{0.693}}{\mathbf{T_{1/2}}}$

or $\frac{4.158 \times 2}{0.693} = t$

or t=12 day

2. **If 15% of the radioactive material decay in 5 days, what would be the percentage of the amount of the original material left after 25 days?**

Solution;

If 15% of the radioactive material decays in 5 days then

$N = (100-15)\%$ of $N_0 = 85\%$ of $N_0 = 0.85\, N_0$

t= 5day

from the decay equation

$N = N_o e^{-\lambda t}$

$0.85\, N_0 = N_o e^{-\lambda t}$

$0.85 = e^{-\lambda t}$

$1/0.85 = e^{\lambda t}$

$\text{Log}_e (1/0.85) = \lambda t$

$\lambda = \frac{\text{Log}_e(1/0.85)}{5} = 0.0325/\text{day}$

Again t=25 days

$\frac{N}{N0} \times 100\% = ?$

From the decay equation $N = N_o e^{-\lambda t}$

$\frac{N}{No} = e^{-\lambda t} = 0.443$

Or $\frac{N}{N0} \times 100\% = 44.3\%$

3. The isotope Ra-226 undergoes α-decay with half-life of 1620 years. What is the activity of 1 g of Ra-226? (Avogadro number=6.023×10^{23} /mole) **(Ans: 3.5×10^{16} disintegration/sec)**

   Hints:
   Half-life($T_{1/2}$) =1620year=$1620\times365\times24\times60\times60=5.1088\times10^{10}$ s
   Mass of substance(m)=1g
   Activity $(\frac{dN}{dt})$=?
   Here $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.1088\times10^{10}} =$
   As we know that,
   226gm of Ra-226= 6.023×10^{23} atom
   1 gm of Ra-226= $\frac{6.023\times10^{23}}{226} = 2.665\times10^{21}$ atom
   therefore, N= 2.665×10^{21}
   Now, $\frac{dN}{dt} = \lambda N =$ **3.5×10^{16} disintegration/sec**

4. At a certain instant a piece of the radioactive material contained 10^{12} atoms. The half-life of the material is 15 days. Calculate the rate of decay after 30 days have elapsed. (**Ans:1.15×10^{10} disintegration/sec)**
5. After a certain lapse of the time, the fraction of the radioactive polonium undecayed found to be 12.5% of initial quantity. what is the duration of this lapse if the half-life of polonium is 139 day?
   (**Ans:417.2 days)**
6. If 4gm of the radioactive material of the half-life period of 10 years disintegrates, find out mean life of the given sample. **(Ans:14.43 year)**
7. Find the half-life of ${}_{92}U^{235,}$ if one gram of it emits 1.24×10^4 alpha particle per second. (Avogadro number=6.023×10^{23} /mole). **Ans: 1.4×10^{17} sec)**
8. Calculate the mass in gram of a radioactive sample Pb-214 having activity 3.7×10^4 decays/sec and half-life of 26.8 min. **(Avogadro number=6.023×10^{23} /mole)**
9. Measurement of on the certain isotopes shows that the decay rate decreases from 8318 decay/min to 3091decay/min in 4 days. What is the half-life of this isotopes?
10. A small volume of solution which contained a radioactive isotope of sodium had an activity of 12000 disintegrations per minute when it was injected into the blood stream of a patient. After 30 hours the activity of 1.0 cm^3 of the blood was found to be 0.5 disintegrations per minute. If the half-life of the sodium isotope is taken as 15 hours. Estimate the volume of the blood in the patient. (6000cc)