WAVE MOTION

A wave is a disturbance from an equilibrium condition that travel with finite velocity from one region to another region of space. Normally the wave is in motion so it is called wave motion.

Types of Wave

(i) Mechanical Waves or Elastic Waves:

The waves which require a material medium for their propagation are called mechanical waves. Eg: Water waves, sound waves, wave on spring etc. for the propagation of mechanical wave, the medium should have three properties; elasticity, inertia and continuity.

(ii) Electromagnetic Waves or Non-mechanical Waves:

The waves which do not require a material medium for their propagation are called electromagnetic waves. Eg: Radio waves, visible and ultraviolet light, microwaves, x-rays etc.

(iii) Matter Waves:

The waves which are associated with the motion of the particles of atomic or subatomic size such as electrons, protons, etc are called matter waves .

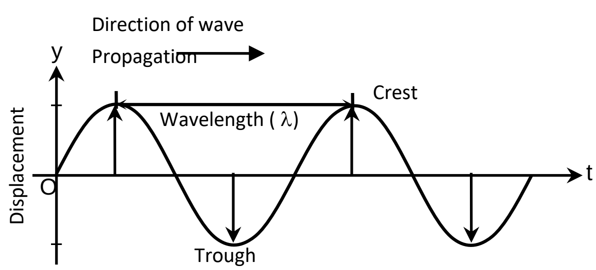
Types of mechanical wave

On the basis of mode of vibration of the particles of the medium, mechanical waves are divided into two types:

(a) Transverse wave and

(b) Longitudinal wave.

a. Transverse wave



The wave motion, in which the particles of the medium vibrate about their mean position at right angle to the direction of propagation of the wave, is called the transverse wave. These waves travel in the form of crest and troughs. Eg. Light wave, vibration of string, rods etc.

b.Longitudinal wave

A wave motion in which the particles of the medium vibrate about their mean position along the direction of the propagation of the wave is known as longitudinal wave. These wave travel in the form of compression and rarefraction.Eg. sound wave in air,wave on springs along length etc.

Some important terms

(a) Displacement (y): The displacement of a particle at any time 't' is the distance it has moved from the mean position at that time. Displacement can be expressed as the function of time as

$$y = A \sin\omega t \quad \text{.....i}$$

Where A is the amplitude and is defined as the maximum displacement of the particle on either side of the mean position.

(b) Particle velocity (*u*):

The particle velocity is that with which the particles of the medium vibrate about their mean position.Taking derivative of equation(i),w.r.t. time *t*, we get

$$u = \frac{dy}{dt} = A\omega \cos \omega t$$

$$\therefore \text{ Particle velocity (u)} = A\omega \cos\omega t$$

C. Wavelength (λ)

It is defined as the distance between adjacent identical parts of a wave(that may be crest or trough) or the linear distance travelled by the vibrating particle in one complete cycle. It is measured in meter (m) in S.I.

d. Wave velocity (v)

The wave velocity is the linear distance travelled per unit time by the wave, which can be written as

as: $v = \frac{\lambda}{T}$

so, $v = \lambda . f$.

e.Time period (T)

It is the time required for the point to complete one full cycle of its motion.It is denoted bt T.

f. Wave frequency (f)

The number of waves produced per second is called frequency of the wave motion. The frequency of a wave is related to the time period as

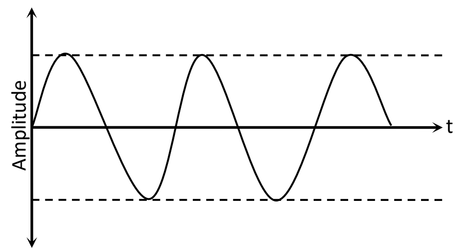
$$f = \frac{1}{T} \text{ (measured in Hz)}$$

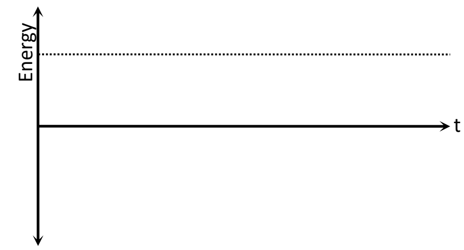
Types of Oscillation

Free Oscillation

The oscillation, in which external forces are absent is called free oscillation. Since, there are no external resistive forces acting on the oscillating body, it oscillates forever with constant amplitude, frequency and energy.

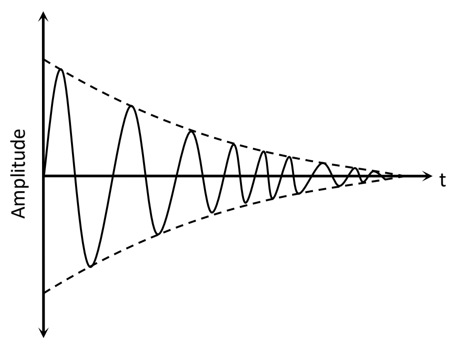
The oscillation of simple pendulum in vacuum is an example of free oscillation.

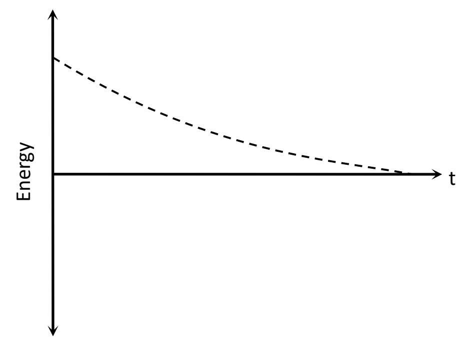




Damped Oscillation

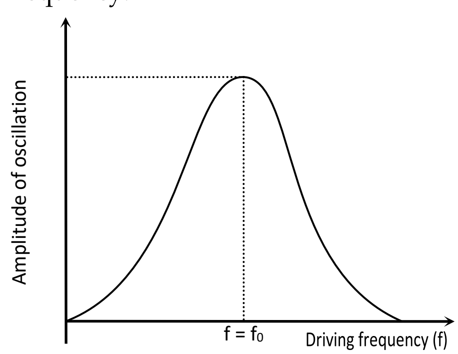
The oscillation, in which amplitude of oscillation gradually decreases and finally becomes zero due to the presence of external resistive force, is called damped oscillation and the process is called damping. The motion of simple pendulum in air is an example of damped oscillation .





Forced Oscillation

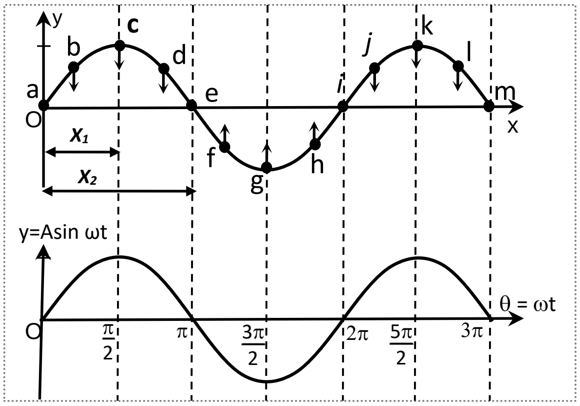
In order to keep the amplitude constant in damped oscillation, the external periodic force must be applied and the oscillation is called forced oscillation. The frequency of this periodic force is called forced frequency.



Resonance

In forced oscillation, when the frequency of external periodic force, coincides with natural frequency of the oscillating body, it oscillates with maximum amplitude. This oscillation is resonant oscillation and the phenomena is called resonance.

Phase, Phase Difference and Path Difference



any two points in the cycle of waveform are said to be in same phase if they are in the same side, equally displaced and oscillating in the same direction. In Fig points (a & i), (b & j are in same phase and these points are separated by an angle 2π, which is called phase difference., a linear distance between two point a and I is λ ,which is called path difference.

relation between path difference and phase difference

let us take any two points ***c*** and ***e***, which are located at a distance x_1 and x_2 at which their phases w.r.t origin are ϕ_1 and ϕ_2; and time to reach the wave from origin to these points is t_1 and t_2 respectively, when

path difference λ , phase difference= 2π

path difference 1, phase difference= 2π/λ

path difference x_1, phase difference= $\phi_1 = (2\pi/\lambda)\, x_1$

Similarly, path difference x_2, phase difference= $\phi_2 = (2\pi/\lambda)\, x_2$

Hence phase difference between two points = $\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$

Where, $\Delta x = x_2 - x_1$ is the difference in distance between two points *c* and *e* is called their path difference.

$$\therefore \text{Phase difference } (\Delta\varphi) = \frac{2\pi}{\lambda} \times \text{path difference } (\Delta x).$$

Progressive wave

A wave which travels continuously in a medium in the same direction without any change in its amplitude and frequency is called a progressive wave or a travelling wave. the particles of the medium execute SHM about their mean position along or perpendicular to the direction of wave propagation according to its longitudinal or transverse nature.

Equation of a progressive wave

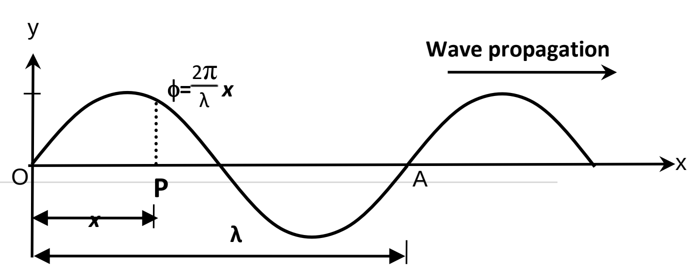


Fig *A progressive wave travelling along positive x-direction*

 © Notes AAYO Team

Let us consider a progressive wave travelling from left to right along the positive x-direction starting from O. the equation of motion of a particle at point O at any instant of time't' is given by,

$$y = a \sin\omega t$$

Where 'a' is the amplitude of wave or particle and ω is the angular velocity of the particles.
Let P is a point at a distance x from O and their phase difference is ϕ, then

$$\phi = \frac{2\pi x}{\lambda}$$

Since the disturbance reaches in later time to the particles to right of O, they start to vibrate after some time w.r.t. the particle at O and the phase lag goes on increasing in this direction. Hence the displacement of a particle at P is given by,

$$y = a \sin(\omega t - \phi)$$
$$= a \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

Here the quantity $\frac{2\pi}{\lambda}$ is a constant for a given wave in a medium is known as propagation constant or wave number and denoted by k. Hence above equation can be written as,

$$y = a \sin(\omega t - kx) \text{..........}i$$

or, $y = a \sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$, Where, v is the velocity of the wave.

$$\therefore \; y = a \sin\frac{2\pi}{\lambda}(vt - x) \quad \text{.........ii}$$

Since $\omega = \frac{2\pi}{T}$, from Eq.i, $y = a \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$

$$\therefore \; y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{.........iii}$$

Equations I, ii and iii represent a plane progressive wave equation.
Similarly, a progressive wave travelling in opposite direction i.e. from right to left
The equation becomes,

$$y = a \sin(\omega t + kx) = a \sin\frac{2\pi}{\lambda}(vt + x) = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right).$$

Principle of Superposition of Waves

"when a large number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the vector sum of the displacements due to the individual waves."

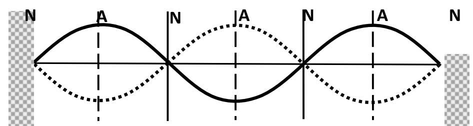
If $y_1, y_2, y_3, \ldots, y_n$ are the displacements due to waves acting separately, then according to the principle of superposition the principle of superposition the resultant displacement y, when all the waves act together is given by,

$y = y_1 + y_2 + y_3 + \ldots + y_n$.. this is the principle of superposition of wave.

 © Notes AAYO Team

Stationary or Standing wave

When two identical progressive waves having same amlitude, frequency and wavelength travelling in same medium with same speed but in opposite direction superimposed with each other, by their superposition ,a new wave is formed which is called standing wave.



A= Antinodes **N**= Nodes

Fig *Shape of the string at different instants.*

Mathematical Treatment of Stationary wave

Let us consider two identical progressive waves of amplitude *a*, frequency *f* and having wavelength λ are travelling in same medium in opposite direction simultaneously. Then these two wave equation are given by,

$$y_1 = a \sin(\omega t + kx) \quad \text{and} \quad y_2 = a \sin(\omega t - kx)$$

Since they are travelling simultaneously,the new wave is formed by their superpositon and resultant displacement of the particles of medium after their superimposition is given by,

$$y = y_1 + y_2$$

$$\text{or,} \quad y = a[\sin(\omega t + kx) + \sin(\omega t - kx)]$$

$$\text{or,} \quad y = 2a \sin\left(\frac{\omega t + kx + \omega t - kx}{2}\right) \cos\left(\frac{\omega t + kx - \omega t + kx}{2}\right)$$

$$\text{or,} \quad y = 2a \cos kx \sin \omega t$$

Since intial waves are sinusoidal wave, their resultant also should be sinusoidal wave. here 2a cos kx = A should be the amplitude of the resultant wave

$$\therefore y = 2a \cos kx \sin \omega t = A \sin \omega t \ \ldots\ldots\ldots\text{i}$$

This is the equation of stationary wave. At a point, $k = \frac{2\pi}{\lambda}$ is constant where as amplitude of the resultant wave, A = 2a cos kx changes with *x,* i.e. amplitude of vibration of the different points is not constant.

a) Position of Nodes:

At nodes amplitude of the resultant wave, i.e. A =2a cos kx should be zero and this is possible only when cos kx = 0.

 © Notes AAYO Team

Or, $\cos\frac{2\pi x}{\lambda} = 0$

or, $\cos\frac{2\pi x}{\lambda} = \cos\left(n + \frac{1}{2}\right)\pi$, where n = 0, 1, 2, ...

or, $\frac{2\pi x}{\lambda} = \left(n + \frac{1}{2}\right)\pi$

or, $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, where n = 0, 1, 2, ...

Hence at the distances $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$, ……. From the boundary, nodes are formed.

Also distance between any two consecutive nodes = $\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$………..ii

b) Position of Antinodes:

At antinodes, the amplitude of the resultant wave, i.e. A =2a cos kx should be maximum and this is possible only when cos kx = $\pm$ 1.

or, $\cos\frac{2\pi x}{\lambda} = \pm 1$

or, $\cos\frac{2\pi x}{\lambda} = \cos n\pi$; where n = 0, 1, 2, ...

or, $\frac{2\pi x}{\lambda} = n\pi$

or, $x = \frac{n\lambda}{2}$; where n = 0, 1, 2, ...

Hence at the distances x = 0, $\frac{\lambda}{2}$, λ, $3\frac{\lambda}{2}$, …. from the boundary, antinodes are formed.

Also distance between any two consecutive antinodes = $\frac{\lambda}{2} - 0 = \frac{\lambda}{2}$……….iii

This concludes that the distance between two consecutive nodes or the distance between two consecutive antinodes is equal to $\frac{\lambda}{2}$ and that distance between node and antinode is $\frac{\lambda}{4}$.

Wave properties

a) Reflection of waves

When the waves are incident on a boundary between two media, part of energy of incident waves returns back into the same medium and this phenomenon is called reflection of waves.

b) Refraction of waves

The bending of wave due to the change in its speed when it travels from one medium to another medium is called refraction of sound.

c) Diffraction of waves

The phenomenon of spreading of the waves when they pass through an aperture or round obstacle is known as diffraction

d) Interference of waves

When two waves of same frequency and and in same phase travel in the same direction along a straight line simultaneously they superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. This phenomenon is called the interference.

Long Question

1. Define progressive waves. Derive an equation to represent this wave.
2. What are stationary waves? Prove that the distance between any two consecutive nodes in a stationery wave is $\frac{\lambda}{2}$.
3. Use the principle of superposition of two waves to find the position of displacement of nodes and antinodes in a standing wave.
4. What is a wave motion? Derive progressive wave equation in a medium.

Short question

1. If you are walking on the moon surface, can you hear the cracking sound behind you? Explain.
2. Distinguish between progressive waves and standing waves.
3. Why echo cannot be heard in a small room?
4. A radio station broadcasts at 800 KHz. What will be the wavelength of the wave?
5. Can two persons on moon hear the sound of each other? Explain.
6. Distinguish between light waves and sound waves.
7. Which types of wave propagate in liquid, explain.
8. Do sound waves under go reflection, refraction and polarization phenomena? Explain.
9. How are stationary waves formed?

Numerical

1. A wave has the equation (x in meters and t in seconds)
   $Y = 0.02 \sin (30 t - 4x)$
   Find i. Its frequency, wavelength and speed.
   ii. The equation of wave with double amplitude but travelling in opposite direction.
2. A wave has speed of 243m/s and a wavelength of 3.27cm. Calculate
   a) Frequency b)time period c)propagation constant d) angular frequency.
3. A progressive wave of frequency 500Hz is travelling with speed of 360m/sec . How far apart are two point s 60° out of phase?
4. A simple harmonic wave is propagating along the –ve x-axis having amplitude of 7cm and frequency of 300 Hz with velocity of 30 m/s. Calculate the displacement, velocity and acceleration of a particle situated 40 cm from the origin at t = 2 sec.

Multiple choice questions

1. The equation of the wave is given by $y = 0.5 \sin(100t-25x)$, the ratio of maximum particle velocity to wave velocity is

a. 25/2 b. 25 c. 4 d. 18/7

2. Two sound waves are respectively represented by $y = a \sin(\omega t-kx)$ and $y = b \cos(\omega t-kx)$. The phase difference between the two waves is

a. $\pi/2$ b. $\pi/4$ c. π d. $3\pi/4$

3. Which parameter of a wave gets affected after superposition?

a. Wavelength b. Direction c. Amplitude d. Frequency

4. Which of the following parameter remain unchanged when the sound wave travels through the medium?.

a. Amplitude b. Frequency c. Velocity d. Wavelength

5. Two sound waves are respectively represented by $y = a \sin(\omega t + \frac{\pi}{6})$ and $y = a \cos \omega t$. The phase difference between the two waves is

a. $\pi/2$ b. $\pi/4$ c. $\pi/3$ d. π

6. Stationary waves of frequency 300 Hz are formed in a medium in which the velocity of sound is 1200 m/s. The distance between the node and the neighboring antinode is

a. 1m b. 2m c. 3m d.4m

7. Equation of stationary wave is $y = 4 \cos \frac{\pi x}{3} \sin 40\pi t$ where x is in meter. The separation between two consecutive nodes is

a. 4m b. 3m c. 6m d. 1.5m

Short question

1. Distinguish between light waves and sound waves.

✍ Light waves and sound waves have the following differences.

Light wave	Sound wave
1. Light wave is a non-mechanical wave which does not require any mechanical medium to transfer from one place to another.	1. Sound wave is a mechanical wave which requires mechanical medium to transfer from one place to another.
2. It is a transverse wave	2. It is a longitudinal wave.
3. It can be polarized.	3. It cannot be polarized.

2. Do sound waves undergo reflection, refraction and polarization phenomena?

✍ Sound wave undergoes reflection and refraction but it does not follow polarization. Sound wave are reflected from surface like walls, big halls etc.During night hearing of sound is more clear than at day time due to the refraction of sound.Since sound wave is longitudinal wave, the vibration of particle occurs to and fro in the direction of the propagation of the wave. The vibrations of such type cannot be limited or controlled by any barriers and so polarization is not possible in them.

3. Write down the difference between progressive wave and stationary wave.

✍ Progressive waves and stationary waves have the following differences.

Progressive Waves	Stationary Waves
1. The disturbance travels forward with a definite velocity.	The disturbance are confined to a particular region.
2. Each particle of the medium executes SHM about its mean position with the same amplitude.	Except nodes, all particles of the medium execute SHM with varying amplitude.
3. There is a continuous change of phase from one particle to the next.	All the particles between two successive nodes vibrate in the same phase, but the phase reverse for particles between next pair of nodes.
4. No particle of the medium is permanently a rest.	The particles of the medium at nodes are permanently at rest.

4. How are stationary wave formed?

When two progressive wave of same wavelength and amplitude travel with the same speed through a medium in opposite direction and superimpose to each other,they give rise to a new wave is called stationary wave.

In stationary wave there are certain points where the amplitude is zero,these points are known as nodes.Midway between the nodes,there are another point where amplitude is maximum.these points are known as antinodes.

5.Frequency is the most fundamental property of wave.Why?

When a wave travel from one medium to another medium,it's velocity and wavelength change but frequency remains same.Hence ,frequency is consireded as to be most fundamental property of wave.

6.We can't hear echo in a small room.why?

The repetation of sound caused by reflection from a cliff, hills or walls of a building is called echo. The minimum distance between sound emitter and reflector must be 17 meter to hear an echo. In a small room, the distance may not be sufficient and hence echo can't be hear.

7. If you are walking on the moon surface, can you hear the cracking sound behind you? Explain.

To travel sound wave from one place to another it needs material medium.Moon has no atmosphere so there is no any material medium.so we can't hear the cracking sound if we walk on the moon's surface.

8. A radio station broadcasts at 800 KHz. If the radio waves travel with a speed of 3×10^8 m/s. What will be the wavelength of the wave?

Wave velocity $(v) = \lambda \times f$,
Where,velocity, $v = c = 3\times 10^8$ m/s
Frequency (f)=800kHz=800000Hz
so, $\lambda = v/f = 3\times 10^8 / 800000 = 375$m.

9. Which types of wave propagate in liquid, explain.